Homework Set # 6 (complete)
Due: 9 a.m. Friday 5/18/07

1. (adapted from Pearl’s classic burglar alarm inference problem) A burglar alarm can be triggered by any of the following three independent events: a burglar, an earthquake, or a false alarm event. Assume that 95% of burglars, 5% of earthquakes, and 100% of false alarm events cause the alarm to go off. Define the following binary random variables: \( b \) (a burglar was present), \( e \) (there was an earthquake), \( f \) (a false alarm event occurred) and \( a \) (the burglar alarm was ringing). Let the prior probabilities be \( P(e = 1) = 0.001, P(b = 1) = 0.002, P(f = 1) = 0.001 \).

(a) Find a factorized expression for \( P(e, b, f, a) \) and draw the corresponding factor graph.

(b) Use the sum product algorithm to find the probability of a burglar conditioned on
   i. knowing that the alarm was ringing
   ii. knowing that the alarm was ringing and an earthquake occurred.

2. Consider a block code with parity check matrix

\[
\begin{bmatrix}
1 & 0 & 1 & 0 & 1 \\
0 & 1 & 0 & 1 & 1
\end{bmatrix}
\]

which is used for communication over a discrete memoryless channel with transition probability \( p(y|x) \) given by

<table>
<thead>
<tr>
<th>( x \backslash y )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.5</td>
<td>0.3</td>
<td>0.15</td>
<td>0.05</td>
</tr>
<tr>
<td>1</td>
<td>0.05</td>
<td>0.15</td>
<td>0.3</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Decode the received vector \((1, 0, 2, 0, 0)\) using the sum product algorithm, stopping if the decoder decision fluctuates between two code vectors.