

Homework Assignment 3 — Solutions

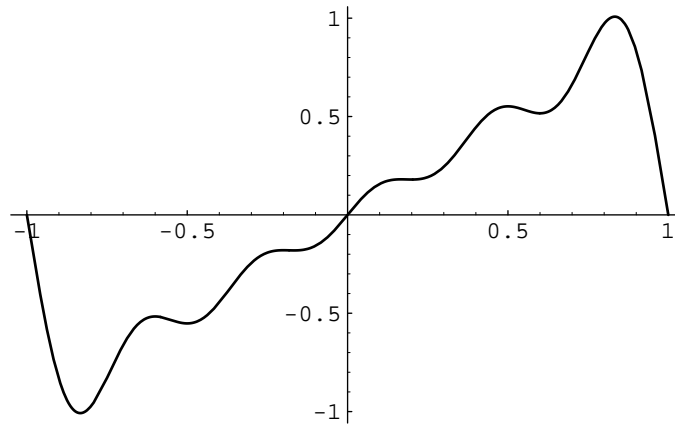
Problem 1. This is a routine calculation. The answer is

$$\begin{aligned}x(t) &= 2 \cos(\omega_0 t + \pi/2) + 4 \cos(5\omega_0 t) \\ &= -2 \sin(\omega_0 t) + 4 \cos(5x\omega_0 t).\end{aligned}$$

Problem 2. Also routine. The answer is

$$x(t) = \sum_{k \geq 1} \frac{(-1)^{k+1}}{k\pi} \sin k\pi t.$$

The sum of the first five terms in this sum looks like this:



Problem 3. The identity obtained is

$$1 + \frac{1}{3} - \frac{1}{5} - \frac{1}{7} + \frac{1}{9} + \frac{1}{11} - \cdots = \frac{\pi}{2\sqrt{2}}.$$

Problem 4.

(a) This was done in class on Oct. 22. But for the record, briefly: with respect to the given norm,

$$\|u_0\|^2 = 2\pi$$

$$\|u_1\|^2 = \pi$$

$$\|u_2\|^2 = \pi$$

Then

$$a_0 = \frac{1}{2\pi} \int_0^{2\pi} x dx = \pi$$

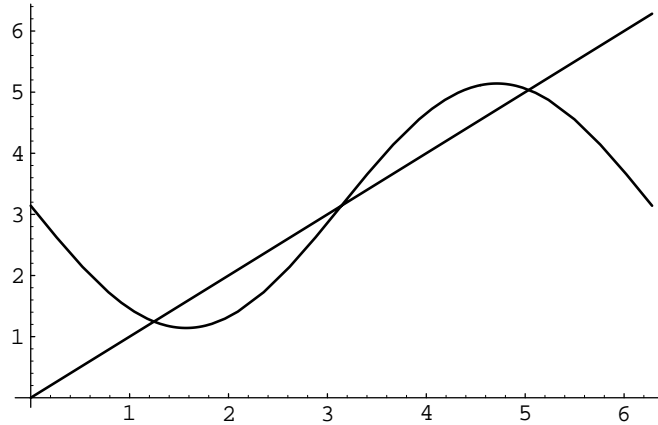
$$a_1 = \frac{1}{\pi} \int_0^{2\pi} x \cos x dx = 0$$

$$a_2 = \frac{1}{\pi} \int_0^{2\pi} x \sin x dx = -2.$$

Thus the best such approximation is

$$\widehat{f}(x) = \pi - 2 \sin x.$$

(b)



(c) A general formula for the error is

$$\begin{aligned} e &= \|f - \widehat{f}\|^2 \\ &= \|f\|^2 - \|\widehat{f}\|^2 \\ &= \|f\|^2 - \sum_n |a_n|^2 \|u_n\|^2. \end{aligned}$$

In our problem, we have $\|f\|^2 = \int_0^{2\pi} x^2 dx = 8\pi^3/3$, $|a_0|^2 \|u_0\|^2 = 4\pi^3$, $|a_1|^2 \|u_1\|^2 = 0$, and $|a_2|^2 \|u_2\|^2 = 4\pi$. The error is then $e = 2\pi^3/3 - 4\pi = 8.11$.