Problem 1. First notice that a simple calculation shows that $Y(z) = z^{-3}X(z^{-1})$.

(a) Thus a pole of $Y(z)$ requires $z^{-3}X(z^{-1}) = \infty$. If $z \neq 0$, this means $X(z^{-1}) = \infty$, i.e., $z^{-1} = 1/2 \pm j/2$ or $z^{-1} = -3/4$, i.e., $z = 1 \pm j$ or $z = -4/3$. At $z = 0$, $z^{-3}$ has a triple pole and $X(z^{-1})$ has a double zero, so $z^{-3}X(z^{-1})$ has a simple pole. In summary: $Y(z)$ has four simple poles: $z = 0$, $1 \pm j$, and $-4/3$.

On the other hand, a zero of $Y(z)$ requires $z^{-3}X(z^{-1}) = 0$. Since $z^{-3}$ is never zero, we must have $X(z^{-1}) = 0$, which happens when $z^{-1} = 0$, i.e., $z = \infty$. In summary: $Y(z)$ has no zeros in the finite $z$-plane. (There is however a simple zero at $z = \infty$.)

(b) Since $x[n]$ is causal, and therefore right-sided, the ROC for $X(z)$ must be $\{z : |z| > 3/4\}$. Since as we observed above, $Y(z) = z^{-3}X(z^{-1})$, it then follows that the ROC for $Y(z)$ is $\{z : 0 < |z| < 4/3\}$.

Problem 2. We have $H_2(\Omega) = a_0 + a_1e^{-j\Omega} + a_2e^{-2j\Omega}$. To minimize

$$\int_{-\pi}^{\pi} |H(e^{j\Omega}) - H_2(e^{j\Omega})|^2 d\Omega,$$

all we have to do is choose $a_0$, $a_1$, and $a_2$ to be the corresponding Fourier series coefficients of the periodic function $H(e^{j\Omega})$. In this case the function $H(e^{j\Omega})$ is a rectangle, and so (e.g. by OW2 Table 5.3),

$$a_0 = \frac{1}{2}\text{sinc}(0) = \frac{1}{2}$$
$$a_1 = \frac{1}{2}\text{sinc}(1/2) = \frac{1}{\pi}$$
$$a_2 = \frac{1}{2}\text{sinc}(1) = 0.$$

Problem 3.

(a) We have $H(z) = \frac{1}{1 - z^{-1} - 4z^{-2}} = \frac{z^2}{z^2 - z - 4}$, which has poles at $z = (1 \pm \sqrt{17})/2 = 2.56, -1.56$. These are both outside the unit circle, so the system is unstable.

(b) Using formula (11.2) from the book, or otherwise, we find that the closed-loop transfer function is $Q(z) = \frac{z^2}{z^2 + (K - 1)z - 4}$.
The coefficient “−4” tells us that the product of the poles of $Q(z)$ is −4, so that both can’t be inside the unit circle. So stabilization is impossible in this case.

(c) In this case,

$$Q(z) = \frac{z^2}{z^2 - z + (K - 4)}.$$  

Using the bilinear transformation $z \leftarrow \frac{1+s}{1-s}$ on the polynomial $z^2 - z + (K - 4)$, we obtain the transformed polynomial

$$P(s) = (K - 2)s^2 - 2(K - 5)s + (K - 4).$$

Dividing by $K - 2$ to make $P(s)$ monic, we need to test

$$P'(s) = s^2 - 2\frac{K - 5}{K - 2} s + \frac{K - 4}{K - 2}$$

for stability. Using the $n = 2$ Hurwitz stability criterion, we see that $P'(s)$ is atable if and only if

$$\frac{K - 5}{K - 2} < 0, \quad \frac{K - 4}{K - 2} > 0.$$ 

After a little algebra, we see that this is equivalent to

$$4 < K < 5.$$ 

Problem 4.

(a) Yes; Yes. (This is really just the basic sampling theorem.)

(b) No, Yes. (Indeed, $x_d[n] = a[n]$ in this case.)

(c) No; No. ($x_d[n] = a[n - 1] + a[n] + a[n + 1]$ in this case, and so for example if $a[n] = \omega^n$, where $\omega = e^{2\pi j/3}$, $x_d[n] = 0$ for all $n$. This means that the signal $a[n]$ cannot be distinguished from the all-zeros signal.)

(d) No; No. ($x_d[n] = a[n - 1] + 3a[n] + a[n + 1]$. Once again we have ISI, and there will always be multiple solutions.)

(e) In the time domain, the condition on $p(t)$ is this: $a[n]$ is always uniquely recoverable from $x(t)$ if and only if $p(0) \neq 0$ and $p(nT_0) = 0$ for $n = \pm 1, \pm 2, \ldots$. 

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