EE32 Solution Set 1

1-)

$$x[n] = \begin{cases} p^{en} & \text{n} \geq 0, \text{and n even} \\ \sin(pn) & \text{n} \geq 0, \text{and n odd} \\ \exp(pn) & \text{n} < 0, \text{and n is prime} \\ 0 & \text{n} < 0, \text{and n is not prime} \end{cases}$$

$$r_R = \overline{\lim_{n\to\infty}} |x[n]|^{1/n} = \overline{\lim_{n\to\infty}} \{p^{en}, \sin(pn)\}^{1/n} = \overline{\lim_{n\to\infty}} \{p^{e}, 0\} = p^{e}$$

$$r_l = \underline{\lim_{n\to-\infty}} |x[n]|^{1/n} = \overline{\lim_{n\to\infty}} \{e^{pn}, 0\}^{-1/n} = \overline{\lim_{n\to\infty}} \{e^{p}, \infty\} = e^{p}$$

Therefore ROC is $= \{z : p^{e} < |z| < e^{p}\}$

2-)

$$h[n] = \begin{cases} (-1)^n / n & \text{if n} \geq 1 \\ 0 & \text{if n} < 0 \end{cases}$$

(a)

$$x[n] = \begin{cases} h[-n] / |h[-n]| & \text{if h[-n]} \neq 0 \\ 0 & \text{if h[-n]} = 0 \end{cases}$$

Then $\quad y[0] = \sum_{k=-\infty}^{\infty} x[k]h[-k] = \sum_{k=-\infty}^{\infty} |h[-k]| = \sum_{k=1}^{\infty} \frac{1}{k} = \infty$; hence the system is not stable.

(b)

Try $x[n] = \begin{cases} (-1)^n & \text{if n} \geq 1 \\ 0 & \text{if n} < 0 \end{cases}$

Then $\quad y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=1}^{n-1} \frac{(-1)^n}{n-k}$; which diverges as $n \to \infty$; hence, the system is not stable.

3-)

1. $x[n]$ is real and right-sided
2. $X(z)$ has exactly 2 poles
3. $X(z)$ has two zeroes at the origin
4. $X(z)$ has a pole at $z = \frac{1}{2} e^{jp/8}$
5. $X(1) = 8/3$

Since the system is real and right sided, poles should occur as conjugates, and since the system has two zeroes at the origin, our function look like this up to a constant term:

$X(z) = \dfrac{cz^2}{(z - \frac{1}{2}e^{jp/8})(z - \frac{1}{2}e^{-jp/8})}$. And also we know that $X(1) = 8/3$, therefore $c = 2$.

Hence, $X(z) = \dfrac{2z^2}{z^2 - 1/2z + 1/4} = \dfrac{8z^2}{4z^2 - 2z + 1}$. And also $x[n]$ is right-sided; so the ROC is $= \{z : \frac{1}{2} < |z|\}$

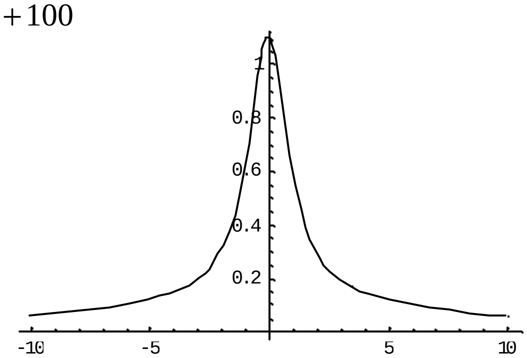
4-)

iv.

$\dfrac{1 - jw/10}{1 + jw} = \dfrac{1 - w^2/10 - 11jw/10}{1 + w^2}$. Hence the angle is $\Phi(w) = \tan^{-1}\left(\dfrac{-11w}{10 - w^2}\right)$.

Therefore, the group delay comes out to be:

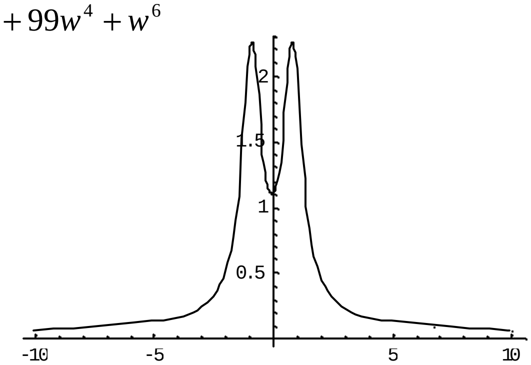
$$-\frac{d\Phi(w)}{dw} = \frac{11w^2 + 110}{w^4 + 101w^2 + 100}$$



vii.

$\dfrac{1 - jw/10}{1 + jw - (jw)^2} = \dfrac{1 - 11w^2/10 - j(11w/10 - w^3/10)}{(1 - w^2)^2 + w^2}$. Hence the angle is $\Phi(w) = \tan^{-1}\left(\dfrac{-11w/10 + w^3/10}{1 - 11w^2/10}\right)$. Therefore, the group delay comes out to be:

$$-\frac{d\Phi(w)}{dw} = \frac{110 + 91w^2 + 11w^4}{100 - 99w^2 + 99w^4 + w^6}$$



My license for the Matlab expired, so I am going to give you the code, you can run it on your own computer:

```
num = [-1/10,1];
den = [1, 1];

func = tf(num, den);
figure(1);
bode(func);

num = [-1/10,1];
den = [1, 1, 1];

func = tf(num, den);
figure(2);
bode(func);
```