Problems to Hand In:

Problem 1. In class on Jan. 30, I suggested a general class of “digital differentiators” with system functions of the form

$$H_N(z) = \sum_{n=1}^{N} \frac{a_n}{2} (z^n - z^{-n}).$$

In this problem, you are supposed to choose the coefficients $(a_1, \ldots, a_N)$ so as to minimize the mean square error

$$\int_{-\pi}^{\pi} |j\Omega - H_N(e^{j\Omega})|^2 d\Omega.$$

(a) Find the appropriate coefficients for $N = 1, 2, 3, 4$.

(b) Is there a general formula for the coefficients found in part (a)?

(c) Plot $(1/j)H_N(e^{j\Omega})$ for $0 < \Omega < \pi$ for $N = 1, 2, 3, 4$, and compare to the “maximum tangency” at $\Omega = 0$ filters derived in class.

Problem 2. In class on Feb. 1, I derived the following “maximum flatness” approximations to the ideal discrete-time differentiating filter with frequency response $H(e^{j\Omega}) = j\Omega$:

$$H_1(e^{j\Omega}) = j \sin \Omega$$
$$H_2(e^{j\Omega}) = j \left( \frac{4}{3} \sin \Omega - \frac{1}{6} \sin 2\Omega \right)$$
$$H_3(e^{j\Omega}) = j \left( \frac{3}{2} \sin \Omega - \frac{3}{10} \sin 2\Omega + \frac{1}{30} \sin 3\Omega \right)$$

Using Matlab, find an expression for the 10th order approximation $H_{10}(e^{j\Omega})$; Plot $|H_1(e^{j\Omega})|$, $|H_2(e^{j\Omega})|$, $|H_3(e^{j\Omega})|$, $|H_{10}(e^{j\Omega})|$, along with $|H(e^{j\Omega})|$ (for $-\pi < \Omega < \pi$) to compare.

Problem 3. Consider discrete-time digital integrating filters defined by the difference equations

$$y[n] = y[n-3] + a_0 x[n] + a_1 x[n-1] + a_1 x[n-2] + a_0 x[n-3]$$

and

$$y[n] = y[n-4] + b_0 x[n] + b_1 x[n-1] + b_2 x[n-2] + b_1 x[n-3] + b_0 x[n-4].$$
Using the ideas to be discussed in class on Feb 4, choose the coefficients $a_i$ and $b_i$ so as to “optimize” (using a maximum flatness criterion) the filters’ performance, and give formulas for the corresponding frequency responses, denoted by $H_3(e^{j\Omega})$ and $H_4(e^{j\Omega})$. Plot $|H_3(e^{j\Omega})|$ and $|H_4(e^{j\Omega})|$ for $-\pi < \Omega < \pi$. 