

Homework Set #3

1. In class we showed that if n_I and n_Q are independent zero-mean Gaussian random variables with variance σ^2 , then the joint distribution of the envelope and phase of

$$A \cos(2\pi f_c t) + n_I \cos(2\pi f_c t) - n_Q \sin(2\pi f_c t) = r \cos(2\pi f_c t + \psi),$$

is given by

$$p_{R,\Psi}(r, \psi) = \frac{r}{2\pi\sigma^2} e^{-\frac{r^2 + A^2 - 2Ar \cos \psi}{2\sigma^2}}.$$

We further computed the marginal distribution of the envelope, r , and showed that it had a Rician density. In this problem we are interested in the marginal density of the phase ψ .

- (a) Show that

$$p_\Psi(\psi) = \frac{e^{-\frac{A^2}{2\sigma^2}}}{2\pi} + \frac{A}{\sigma} \cdot \frac{\cos \psi}{\sqrt{8\pi}} \cdot e^{-\frac{A^2}{2\sigma^2} \sin^2 \psi} \operatorname{erfc}\left(-\frac{A}{\sigma} \cos \psi\right),$$

where $\operatorname{erfc}(\cdot)$ is the *complementary error function* defined as

$$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-u^2} du.$$

- (b) Using the function $\operatorname{erfc}(\cdot)$ in MATLAB plot the marginal density for the values $\frac{A}{\sigma} = 0.01, 0.1, 1$ on a single graph, as well as for the values $\frac{A}{\sigma} = 1, 5, 20$ on another single graph. Briefly comment on the results.

2. Show that

$$\int_0^\infty \frac{r^3}{\sigma^2} e^{-\frac{r^2 + A^2}{2\sigma^2}} I_0\left(\frac{Ar}{\sigma^2}\right) dr = A^2 + 2\sigma^2.$$

(Hint: Do not attempt to directly compute the integral. Try to find a physical interpretation.)