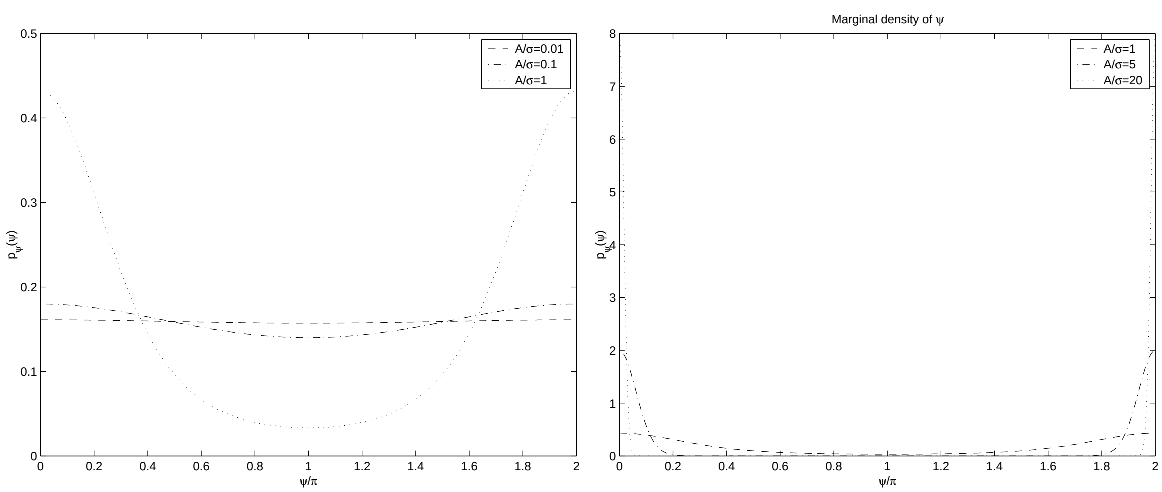
EE160 Handout #10
February 14, 2008

Solutions to Homework Set #3

1. (a)

$$
\begin{aligned}
p_\Psi(\psi) &= \int_{-\infty}^{\infty} p_{R,\Psi}(r,\psi)dr \\
&= \int_0^{\infty} \frac{r}{2\pi\sigma^2} e^{-\frac{r^2+A^2-2Ar\cos\psi}{2\sigma^2}} dr \\
&= \frac{e^{-\frac{A^2\sin^2\psi}{2\sigma^2}}}{2\pi\sigma^2} \int_0^{\infty} r e^{-\frac{(r-A\cos\psi)^2}{2\sigma^2}} dr \\
&= \frac{e^{-\frac{A^2\sin^2\psi}{2\sigma^2}}}{2\pi\sigma^2} \int_{-A\cos\psi}^{\infty} (t + A\cos\psi) e^{-\frac{t^2}{2\sigma^2}} dt \\
&= \frac{e^{-\frac{A^2\sin^2\psi}{2\sigma^2}}}{2\pi\sigma^2} A\sigma\sqrt{2}\cos\psi \int_{-\frac{A\cos\psi}{\sigma\sqrt{2}}}^{\infty} e^{-u^2} du + \frac{e^{-\frac{A^2\sin^2\psi}{2\sigma^2}}}{2\pi\sigma^2} \frac{1}{2} \int_{A^2\cos^2\psi}^{\infty} e^{-\frac{u}{2\sigma^2}} du \\
&= \frac{A}{\sigma} \frac{\cos\psi}{\sqrt{8\pi}} e^{-\frac{A^2\sin^2\psi}{2\sigma^2}} \operatorname{erfc}\left(-\frac{A\cos\psi}{\sigma\sqrt{2}}\right) + \frac{e^{-\frac{A^2}{2\sigma^2}}}{2\pi}
\end{aligned}
$$



(b) The marginals are as plotted above. We note that for small values of $\frac{A}{\sigma}$ we get a uniform pdf in $[0, 2\pi)$ and for larger values the phase becomes zero.

2. Let r be a random variable obtained as $r^2 = x^2 + y^2$ where x and y are independent Gaussian random variables with identical variance σ^2 and means 0 and A respectively.

Then r is Rician distributed as $\frac{r}{\sigma^2}e^{-\frac{r^2+A^2}{2\sigma^2}}I_0(\frac{Ar}{\sigma^2})$. Hence $\int_0^\infty \frac{r^3}{\sigma^2}e^{-\frac{r^2+A^2}{2\sigma^2}}I_0(\frac{Ar}{\sigma^2})dr$ is simply Er^2. Hence we have

$$\begin{aligned}
\int_0^\infty \frac{r^3}{\sigma^2}e^{-\frac{r^2+A^2}{2\sigma^2}}I_0\left(\frac{Ar}{\sigma^2}\right)dr &= Er^2 \\
&= E(x^2+y^2) \\
&= Ex^2+Ey^2 \\
&= \sigma^2+(\sigma^2+A^2) \\
&= A^2+2\sigma^2
\end{aligned}$$