EE160 Handout #11

February 14, 2008

Solutions to Homework Set #4

1.
$$v_2(t) = (A_c \cos 2\pi f_c t + m(t))\left[\frac{1}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}\cos(2\pi f_c t(2n-1))\right] \quad (1)$$

(a) Since the carrier is at f_c the AM wave component is $\frac{1}{2}A_c \cos(2\pi f_c t)(1 + \frac{4m(t)}{\pi A_c})$

(b) The unwanted components are those at all the other multiples of f_c (including dc) i.e.

$$\frac{2}{\pi}A_c \cos(2\pi f_c t)\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}\cos(2\pi f_c t(2n-1)) + m(t)\left(\frac{1}{2} + \frac{2}{\pi}\sum_{n=2}^{\infty}\frac{(-1)^{n-1}}{2n-1}\cos(2\pi f_c t(2n-1))\right) \quad (2)$$

2. Let $m(t) \rightleftharpoons M(f)$. Let $M(f)u(f) = M^+(f)$ and $M(f)u(-f) = M^-(f)$. Then

$$\begin{aligned}
\frac{1}{2}m(t)\cos 2\pi f_c t &\rightleftharpoons \frac{1}{4}(M(f-f_c) + M(f+f_c)) \\
&= \frac{1}{4}(M^+(f-f_c) + M^-(f-f_c) + M^+(f+f_c) + M^-(f+f_c))
\end{aligned}$$

and

$$\begin{aligned}
\frac{1}{2}\hat{m}(t)\sin 2\pi f_c t &\rightleftharpoons \hat{M}(f) * \frac{1}{4j}(\delta(f-f_c) - \delta(f+f_c)) \\
&= (-jM^+(f) + jM^-(f)) * \frac{1}{4j}(\delta(f-f_c) - \delta(f+f_c)) \\
&= \frac{1}{4}(-M^+(f-f_c) + M^-(f-f_c) + M^+(f+f_c) - M^-(f+f_c))
\end{aligned}$$

Hence

$$s(t) \rightleftharpoons \frac{1}{2}(M^-(f-f_c) + M^+(f+f_c)) \quad (3)$$

or

$$s(t) \rightleftharpoons \frac{1}{2}(M^+(f-f_c) + M^-(f+f_c)) \quad (4)$$

giving lower and upper sidebands respectively.

3. From Figure 2.12 we have

$$s(t) \rightleftharpoons \frac{1}{2}(M(f-f_c) + M(f+f_c))H(f) \quad (5)$$

But from Eqn. 2.15 we have

$$\begin{aligned} s(t) &\rightleftharpoons \frac{1}{2}(M(f-f_c)+M(f+f_c)) \pm \frac{1}{2j}(M'(f-f_c)-M'(f+f_c)) \\ &= \frac{1}{2}(M(f-f_c)+M(f+f_c)) \pm \frac{1}{2j}(M(f-f_c)H_Q(f-f_c)-M(f+f_c)H_Q(f+f_c)) \\ &= \frac{1}{2}[M(f-f_c)(1 \pm \frac{1}{j}H_Q(f-f_c)) + M(f+f_c)(1 \mp \frac{1}{j}H_Q(f+f_c))] \end{aligned}$$

Comparing we have

$$\begin{aligned} H(f) &= 1 \pm \frac{1}{j}H_Q(f-f_c) \\ H(f) &= 1 \mp \frac{1}{j}H_Q(f+f_c) \end{aligned}$$

which implies

$$\begin{aligned} H(f+f_c) &= 1 \pm \frac{1}{j}H_Q(f) \\ H(f-f_c) &= 1 \mp \frac{1}{j}H_Q(f) \end{aligned}$$

for $-W \le f \le W$. Taking the difference we get

$$H_Q(f) = j[H(f-f_c) - H(f+f_c)] \tag{6}$$

for $-W \le f \le W$ upto scale.

4. The SSB-SC signal is given by

$$s(t) = CA_c\cos(2\pi f_c t)m(t) + CA_c\hat{m}(t)\sin(2\pi f_c t) \tag{7}$$

If we have $\int_{-W}^{W} S_M(f)df = P$ then we also have $\int_{-W}^{W} S_{\hat{M}}(f)df = P$.

(a) The average power of the SSB-SC modulated signal component $s(t)$ is $C^2A_c^2P/2 + C^2A_c^2P/2$ since the two components are orthogonal. With a noise spectral density of $N_0/2$, the average noise power in the message bandwidth W is WN_0. Hence $(SNR)_{C,SSB} = \frac{C^2A_c^2P}{WN_0}$.

(b) At the output

$$\begin{aligned} v(t) &= (s(t)+n(t))cos(2\pi f_c t) \\ &= \frac{1}{2}CA_c m(t) + \frac{1}{2}n_I(t) + \text{high frequency terms} \end{aligned}$$

At the output of the lowpass filter we have

$$y(t) = \frac{1}{2}CA_c m(t) + \frac{1}{2}n_I(t)$$

The average power of the signal compnent is $C^2A_c^2P/4$. the band-pass filter has bandwidth W. Hence the average noise power due to $n_I(t)/2$ is $\frac{1}{4}WN_0$. Therefore $(SNR)_{O,SSB} = \frac{C^2A_c^2P}{WN_0}$

The figure of merit

$$\left.\frac{(SNR)_O}{(SNR)_C}\right|_{SSB-SC} = 1 \tag{8}$$

as in the case of DSB-SC.