

Homework Set #6

1. Problem 3.11(e) in Haykin.
2. Problem 3.23 in Haykin.
3. Suppose that m is a random variable uniformly distributed between $-A$ and A . Show that, for any L , the optimal L -level quantizer is the uniform quantizer.
4. Consider the random variable m of Problem 3 and suppose that we quantize it using a three-bit quantizer ($L = 8$).
 - (a) Suppose that we encode the ordered quantization intervals \mathcal{I}_k , $k = 0, \dots, 7$, simply by the representation of k as a three-bit binary number. (See Table 3.2 on page 204 of the book, for example.) The resulting binary sequence is then transmitted across some communications channel where the probability of making an error on a single bit is P_e . Compute the resulting expected mean-square distortion of the reconstructed signal, assuming that the probability of making more than one bit error for every three bits is negligible.
 - (b) Give a three-bit Gray encoder, i.e., one for which the representation of all adjacent intervals differ in only one bit.
 - (c) Compute the resulting mean-square distortion of the reconstructed signal using the Gray encoder and compare the result with part (a).
5. Consider a random variable m whose probability density function is given by

$$p_M(m) = \begin{cases} 1 - |m| & |m| \leq 1 \\ 0 & \text{otherwise} \end{cases} .$$

Find the optimal two-bit Lloyd-Max quantizer for mean-square distortion. How does the resulting mean-square distortion compare to that of a two-bit uniform quantizer?

6. Suppose that we sample a random process with power spectral density

$$S_m(f) = \begin{cases} 1 & |f| \leq W \\ 0 & \text{otherwise} \end{cases} ,$$

at twice the Nyquist rate and use a linear predictive coder, quantized to R bits per sample. What are the output signal to noise ratios when the prediction is based on 1, 2 and 3 previous samples each? How does this compare with the signal to quantization-noise ratio of a system that takes samples at the Nyquist rate, uses no linear prediction, but quantizes to $2R$ bits per sample.