Homework Set #7

1. Suppose that we sample a random process with power spectral density

\[ S_m(f) = \begin{cases} 
1 & |f| \leq W \\
0 & \text{otherwise} 
\end{cases}, \]

at twice the Nyquist rate and use a linear predictive coder, quantized to \( R \) bits per sample. What are the output signal to noise ratios when the prediction is based on 1, 2 and 3 previous samples each? How does this compare with the signal to quantization-noise ratio of a system that takes samples at the Nyquist rate, uses no linear prediction, but quantizes to \( 2R \) bits per sample.

2. Haykin’s problems 4.9 and 4.11.

3. The output of the receive filter in a PCM system is given by

\[ y(t) = \sum_{k=-\infty}^{\infty} a_k p(t - kT_b), \]

where the \( \{a_k\} \) are iid random variables that take on the values \( a_k = 1 \) and \( a_k = -1 \) with equal probability. Suppose that the receiver is not quite synchronized with the transmitter, so that \( y(t) \) is sampled at time instants \( t_i = iT_b + \delta t \), rather than at \( t_i = iT_b \).

(a) Show that the resulting mean-square-error at the sampled output is

\[ E(y(iT_b + \delta) - a_i)^2 = (1 - p(\delta t))^2 + \sum_{k \neq 0} p^2(\delta t + kT_b). \]

(b) Define the “time-normalized” pulse function \( q(t) = p(tT_b) \), and hence show that, if \( \delta t/T_b \) is small, then to first order

\[ E(y(iT_b + \delta) - a_i)^2 \approx \left( \frac{\delta t}{T_b} \right)^2 \sum_{k=-\infty}^{\infty} (q'(k))^2. \]

(c) Compute and plot the factor \( \sum_{k=-\infty}^{\infty} (q'(k))^2 \) as a function of \( \alpha \) for the raised cosine signal. (Eq. 4.62 in Haykin.) How many db’s of improvement do we get for \( \alpha = 1 \), compared to \( \alpha = 0 \) (which is the standard sinc pulse)?

(d) Compute the factor \( \sum_{k=-\infty}^{\infty} (q'(k))^2 \) for the correlative level pulse (of Haykin’s Eq. (4.70)). How does it compare to the sinc pulse?

4. Haykin’s problem 4.32.