EE160 Handout #14
February 15 2007

Midterm - Winter 2007

- There are 5 problems for a total of 30 points.
- The problems may be of different difficulty, so schedule your time accordingly.
- The exam is open-book, open course-notes, etc. Any result from the book, handouts and/or homework solutions can be quoted without proof.
- Sub-problems within a problem are independent, so if you can solve part (c) of a problem, but not part (b), say, do so!

GOOD LUCK!!!

1. A zero-mean stationary discrete-time process $\{x_i\}_{i=-\infty}^{\infty}$ is said to have memory one if

$$Ex_i x_{i-L} = 0, \qquad \forall L > 1.$$

   (a) (*4 points*) Show that

$$|Ex_i x_{i-1}| \leq \frac{1}{2} Ex_i^2.$$

   Hint: The power spectral density of a discrete-time process is defined as

$$S_X(e^{j\omega}) = \sum_{i=-\infty}^{\infty} R_x(i) e^{-j\omega i}.$$

   (b) (*4 points*) Show that $\{x_i\}_{i=-\infty}^{\infty}$ is ergodic in the mean.

2. Consider the process $x(\cdot)$ obtained from

$$\frac{dx(t)}{dt} = ax(t) + u(t), \qquad t \geq 0$$

where $a < 0$ and $u(\cdot)$ is zero-mean white noise with power spectral density $\frac{\eta}{2}$. Assuming the initial value of $x(t)$ is $X(0) = x_0$, this implies that

$$x(t) = e^{at} x_0 + \int_0^t e^{a(t-\tau)} u(\tau) d\tau.$$

   (a) (*2 points*) Assume $x_0 = 0$. What is the mean of $x(t)$?

   (b) (*3 points*) Assuming again that $x_0 = 0$, is $x(\cdot)$ wide-sense stationary?

   (c) (*3 points*) Assume x_0 is random, independent of $u(\cdot)$ with zero-mean and variance σ^2. Show that $x(\cdot)$ is wide-sense stationary if, and only if,

$$\sigma^2 a + \frac{\eta}{4} = 0.$$

3. Since the DSB-SC signal $x(t) = m(t)\cos(2\pi f_c t + \theta)$ occupies twice the bandwidth of the message $m(t)$, it is tempting to transmit two independent messages $m_1(t)$ and $m_2(t)$ over the same bandwidth via

$$x(t) = m_1(t)\cos(2\pi f_c t + \theta) - m_2(t)\sin(2\pi f_c t + \theta).$$

   (a) (*2 points*) Design a receiver that demodulates both messages $m_1(t)$ and $m_2(t)$.

   (b) (*2 points*) If the receiver is not synchronized with the transmitter, i.e., if the local oscillator at the receiver has a phase difference with the carrier frequency generated by the transmitter, is it possible to demodulate the messages $m_1(t)$ and $m_2(t)$? In other words, is a receiver structure like the Costas receiver possible? Explain.

4. The single sideband version of angle modulation is given by

$$x(t) = e^{-\hat{\phi}(t)} \cos(2\pi f_c t + \phi(t)),$$

where $\hat{\phi}(t)$ is the Hilbert transform of the phase function $\phi(t)$ and f_c is the carrier frequency. It can indeed be shown that the power spectral density function of $x(\cdot)$ is zero for all frequencies $|f| < f_c$, however, we shall not do so here.

(a) (*3 points*) Design a receiver to recover the signal $m(t)$ from the above $x(t)$ when

$$\phi(t) = 2\pi k_f \int_{-\infty}^{t} m(\tau) d\tau.$$

(b) (*2 points*) Briefly describe what the effect of noise is on the performance of your proposed receiver.

5. Let $m(\cdot)$ be a zero-mean wide sense stationary process with autocorrelation function $R_m(\tau)$.

(a) (*3 points*) Suppose $m(\cdot)$ is DSB-SC-modulated to yield

$$x(t) = m(t) \cos (2\pi f_c t + \theta),$$

where θ is a random phase, independent of $m(\cdot)$ and uniformly distributed on $[0, 2\pi]$. Suppose further that $x(t)$ is passed through an FM demodulator (differentiator, followed by envelope detector). What is the output of the demodulator?

(b) (*2 points*) Suppose now that $m(\cdot)$ is FM-modulated to yield

$$x(t) = \cos \left(2\pi f_c t + \int_{-\infty}^{t} m(\tau) d\tau + \theta\right).$$

If now $x(t)$ is input to a coherent DSB-SC demodulator, what is the output?