Noise Optimized Eigenfilter Design Of Time-Domain Equalizers For DMT Systems

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***Abstract* – The design of time-domain equalizers or TEQs for discrete multitone modulation (DMT) systems has recently received much attention. In this paper, we present a generalization of one such design method which takes into account the noise observed in a DMT channel. Furthermore, we show how this generalization can be used for the design of fractionally spaced equalizers or FSEs. Experimental results are presented showing that our design method performs better than other known techniques.** [1]

I. Introduction

One problem which has been of great interest in recent years has been the design of time-domain equalizers or TEQs for discrete multitone modulation or DMT systems [1, 4, 6]. Due to the long impulse response of typical channels encountered in DMT systems such as twisted pair telephone lines [7], TEQs are necessary to *shorten* the overall channel response to one sample more than the length of the cyclic prefix used.

Many of the methods proposed for the design of such TEQs deal with the design of the effective channel (i.e. the cascade of the channel and equalizer) [1, 4] and not the equalizer coefficients directly. In these methods, the equalizer coefficients must then be chosen to best fit the desired optimal effective channel. Recently, however, a new method was proposed [6], which deals directly with the equalizer coefficients. The objective of this method is to minimize the delay spread of the overall channel. This method was shown to localize the temporal spread of the effective channel more so than other methods and was shown to be less sensitive to synchronization errors as well. However, this method does not take into account the noise present in the system.

In this paper, we consider generalizing the method of [6] to include the effects due to noise. The optimum equalizer filter coefficients will be found to be related to the components of an eigenvector of a particular matrix. Furthermore, we will show that our results can be extended for the design of fractionally spaced equalizers or FSEs. Although FSEs have not been used as TEQs for DMT systems, the results obtained here give merit to their possible future use in such systems.

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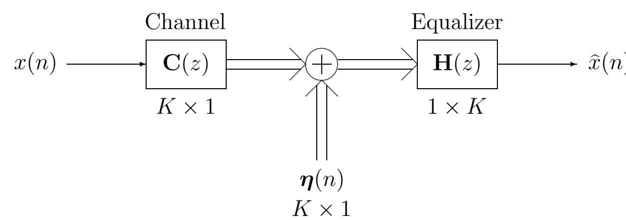

Fig. 1. SIMO-MISO channel and equalizer model.

A. Notations

All notations are as in [11]. In particular, any transfer function $H(z)$ can be decomposed into one of the following types of polyphase decompositions for any positive integer M.

$$H(z) = \sum_{k=0}^{M-1} z^{-k} E_k(z^M) \quad \text{(Type I)}$$

$$H(z) = \sum_{k=0}^{M-1} z^{k} R_k(z^M) \quad \text{(Type II)}$$

where we have,

$$e_k(n) = h(Mn+k)\,,\ r_k(n) = h(Mn-k)\,,\ 0 \le k \le M-1\,.$$

II. The TEQ Design Problem

Suppose that we have the single-input multiple-output (SIMO) channel and multiple-input single-output (MISO) equalizer model as shown in Figure 1. When $K = 1$ we obtain the classical symbol spaced equalizer (SSE) model. In Section IV we will show that the fractionally spaced equalizer (FSE) is a special case of this model. We make the following assumptions here.

- The channel $\mathbf{C}(z)$ is FIR of length L_c.
- The equalizer $\mathbf{H}(z)$ is FIR of length L_e.
- The noise vector sequence $\boldsymbol{\eta}(n)$ is a WSS random process with psd matrix $\mathbf{S}_{\boldsymbol{\eta}\boldsymbol{\eta}}(e^{j\omega})$.

For simplicity, we denote the impulse responses of $\mathbf{C}(z)$ and $\mathbf{H}(z)$ by $\mathbf{c}(n)$ and $\mathbf{h}(n)$, respectively. The effective channel is

$c_{\text{eff}}(n) = \mathbf{h}(n) * \mathbf{c}(n)$ and has length $L_c + L_e - 1$. Note that the output $\widehat{x}(n)$ can be expressed as follows.

$$\widehat{x}(n) = x_s(n) + w(n)$$

where $x_s(n)$ and $w(n)$ are, respectively, the output signal and output noise sequences given by the following.

$$\begin{aligned} x_s(n) &= c_{\text{eff}}(n) * x(n) \\ w(n) &= \mathbf{h}(n) * \boldsymbol{\eta}(n) \end{aligned}$$

We wish to choose the coefficients of the equalizer to accomplish the following goals.

- Shorten the effective channel $c_{\text{eff}}(n)$.
- Minimize the noise power σ_w^2 with respect to the signal power $\sigma_{x_s}^2$.

To that end, we propose to choose the coefficients of $\mathbf{h}(n)$ to minimize the following objective function J.

$$J \triangleq \alpha J_{\text{short}} + (1-\alpha) J_{\text{noise}}\,,\ 0 \le \alpha \le 1 \tag{1}$$

where J_{short} and J_{noise} are defined as follows.

$$J_{\text{short}} \triangleq \frac{\displaystyle\sum_n f(n - n_{\text{mid}}) |c_{\text{eff}}(n)|^2}{\displaystyle\sum_n |c_{\text{eff}}(n)|^2} \tag{2}$$

$$J_{\text{noise}} \triangleq \frac{\sigma_w^2}{\sigma_{x_s}^2} = \frac{\sigma_w^2}{\sigma_x^2 \displaystyle\sum_n |c_{\text{eff}}(n)|^2} \tag{3}$$

Here, the quantity J_{short} represents a channel shortening objective function whereas J_{noise} is the noise-to-signal ratio (under the assumption that the input signal $x(n)$ is white). The parameter n_{mid} denotes the desired midpoint or "centroid" of the effective channel. In addition, the function $f(n)$ is a "penalty" function which is any nonnegative function used to penalize values of $c_{\text{eff}}(n)$ that are away from $n = n_{\text{mid}}$. Examples of penalty functions that we will consider here are shown below.

$$\begin{aligned} f(n) &= n^2 \quad \text{(Quadratic)} \\ f(n) &= |n| \quad \text{(Linear)} \end{aligned}$$

The special case in which $K = 1$, $\alpha = 1$, and $f(n) = n^2$ was analyzed previously by Schur and Speidel [6]. Since the primary design goal of a TEQ for DMT systems is to shorten the overall channel to one sample more than the length of the cyclic prefix, we will also consider the following penalty function.

$$f(n) = \begin{cases} 0\,, & n \in \left[-\frac{N_{\text{CP}}}{2}, \frac{N_{\text{CP}}}{2}\right] \\ 1\,, & \text{otherwise} \end{cases} \tag{4}$$

where N_{CP} is the length of the cyclic prefix (assumed to be even here). Note that J is a *convex combination* of the objective functions J_{short} and J_{noise}, and that $J \ge 0$. Here, the parameter α represents a tradeoff parameter between shortening the effective channel and minimizing the output noise-to-signal ratio. We will now proceed to analyze the objective function J.

III. Analysis of the Objective Function J

First we will analyze J_{short}. To do so, we define the following vectors and matrices.

$$\begin{aligned} \mathbf{c}_{\text{eff}} &\triangleq \begin{bmatrix} c_{\text{eff}}(0) & c_{\text{eff}}(1) & \cdots & c_{\text{eff}}(L_c + L_e - 2) \end{bmatrix} \\ \mathbf{h} &\triangleq \begin{bmatrix} \mathbf{h}(0) & \mathbf{h}(1) & \cdots & \mathbf{h}(L_e - 1) \end{bmatrix} \end{aligned}$$

$$\mathbf{C} \triangleq \begin{bmatrix} \mathbf{c}(0) & \mathbf{c}(1) & \cdots & \mathbf{c}(L_c-1) & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{c}(0) & \mathbf{c}(1) & \cdots & \mathbf{c}(L_c-1) & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{c}(0) & \mathbf{c}(1) & \cdots & \mathbf{c}(L_c-1) \end{bmatrix}$$

$$\boldsymbol{\Lambda} \triangleq \begin{bmatrix} \sqrt{f(0-n_{\text{mid}})} & 0 & \cdots & 0 \\ 0 & \sqrt{f(1-n_{\text{mid}})} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \sqrt{f((L_c+L_e-2)-n_{\text{mid}})} \end{bmatrix}$$

These quantities have the following sizes.

- $\mathbf{c}_{\text{eff}} : 1 \times (L_c + L_e - 1)$
- $\mathbf{h} : 1 \times KL_e$
- $\mathbf{C} : KL_e \times (L_c + L_e - 1)$
- $\boldsymbol{\Lambda} : (L_c + L_e - 1) \times (L_c + L_e - 1)$

Note that $\mathbf{c}_{\text{eff}}$ is given by convolution to be the following.

$$\mathbf{c}_{\text{eff}} = \mathbf{hC}$$

Using this and the definition of J_{short} given in (2), we have,

$$J_{\text{short}} = \frac{(\mathbf{c}_{\text{eff}}\boldsymbol{\Lambda})(\mathbf{c}_{\text{eff}}\boldsymbol{\Lambda})^\dagger}{\mathbf{c}_{\text{eff}}\mathbf{c}_{\text{eff}}^\dagger} = \frac{\mathbf{hC}\boldsymbol{\Lambda}\boldsymbol{\Lambda}^\dagger\mathbf{C}^\dagger\mathbf{h}^\dagger}{\mathbf{hCC}^\dagger\mathbf{h}^\dagger} \tag{5}$$

Now, assuming $KL_e \le L_c + L_e - 1$ and that $\mathbf{C}$ has a full rank of KL_e, then the matrix $\mathbf{A} \triangleq \mathbf{CC}^\dagger$ is strictly positive definite. As such, there exists a *Cholesky decomposition* [5] of $\mathbf{A}$ as,

$$\mathbf{A} = \mathbf{U}^\dagger\mathbf{U}$$

where $\mathbf{U}$ is a $KL_e \times KL_e$ nonsingular upper triangular matrix. Using this decomposition, we can express J_{short} as a Rayleigh quotient [5]. Defining the $KL_e \times 1$ column vector $\mathbf{v}$ as follows,

$$\mathbf{v} \triangleq \mathbf{Uh}^\dagger \iff \mathbf{h} = \mathbf{v}^\dagger(\mathbf{U}^{-1})^\dagger \tag{6}$$

we have from (5),

$$J_{\text{short}} = \frac{\mathbf{v}^\dagger(\mathbf{U}^{-1})^\dagger\mathbf{C}\boldsymbol{\Lambda}\boldsymbol{\Lambda}^\dagger\mathbf{C}^\dagger(\mathbf{U}^{-1})\mathbf{v}}{\mathbf{v}^\dagger\mathbf{v}} = \frac{\mathbf{v}^\dagger\mathbf{P}\mathbf{v}}{\mathbf{v}^\dagger\mathbf{v}} \tag{7}$$

where $\mathbf{P} \triangleq (\mathbf{U}^{-1})^\dagger\mathbf{C}\boldsymbol{\Lambda}\boldsymbol{\Lambda}^\dagger\mathbf{C}^\dagger(\mathbf{U}^{-1})$. Evidently, $\mathbf{P}$ is Hermitian and thus J_{short} has been expressed as a Rayleigh quotient in terms of the vector $\mathbf{v}$. We now proceed to analyze the noise objective function J_{noise}. From (3), we have,

$$J_{\text{noise}} = \frac{\sigma_w^2}{\sigma_x^2\mathbf{hCC}^\dagger\mathbf{h}^\dagger} \tag{8}$$

where σ_w^2 is given by,

$$\sigma_w^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} S_{ww}(e^{j\omega})\,d\omega$$

As $w(n) = \mathbf{h}(n) * \boldsymbol{\eta}(n)$, we have the following.

$$\begin{aligned} S_{ww}(e^{j\omega}) &= \mathbf{H}(e^{j\omega})\mathbf{S}_{\boldsymbol{\eta}\boldsymbol{\eta}}(e^{j\omega})\mathbf{H}^\dagger(e^{j\omega}) \\ &= \sum_{m,n}\mathbf{h}(m)\left[\mathbf{S}_{\boldsymbol{\eta}\boldsymbol{\eta}}(e^{j\omega})e^{j\omega(n-m)}\right]\mathbf{h}^\dagger(n) \end{aligned}$$

$$\sigma_w^2 = \sum_{m,n}\mathbf{h}(m)\underbrace{\left[\frac{1}{2\pi}\int_{-\pi}^{\pi}\mathbf{S}_{\boldsymbol{\eta}\boldsymbol{\eta}}(e^{j\omega})e^{j\omega(n-m)}\,d\omega\right]}_{\mathbf{R}_{\boldsymbol{\eta}\boldsymbol{\eta}}(n-m)}\mathbf{h}^\dagger(n)$$

Defining the $KL_e \times KL_e$ matrix $\mathbf{R}_{\boldsymbol{\eta}}$ as follows,

$$\mathbf{R}_{\boldsymbol{\eta}} \triangleq \begin{bmatrix} \mathbf{R}_{\boldsymbol{\eta}\boldsymbol{\eta}}(0) & \mathbf{R}_{\boldsymbol{\eta}\boldsymbol{\eta}}(1) & \cdots & \mathbf{R}_{\boldsymbol{\eta}\boldsymbol{\eta}}(L_e-1) \\ \mathbf{R}_{\boldsymbol{\eta}\boldsymbol{\eta}}(-1) & \mathbf{R}_{\boldsymbol{\eta}\boldsymbol{\eta}}(0) & \ddots & \vdots \\ \vdots & \ddots & \ddots & \mathbf{R}_{\boldsymbol{\eta}\boldsymbol{\eta}}(1) \\ \mathbf{R}_{\boldsymbol{\eta}\boldsymbol{\eta}}(-(L_e-1)) & \cdots & \mathbf{R}_{\boldsymbol{\eta}\boldsymbol{\eta}}(-1) & \mathbf{R}_{\boldsymbol{\eta}\boldsymbol{\eta}}(0) \end{bmatrix}$$

we can express σ_w^2 in terms of the vector $\mathbf{h}$ as shown below.

$$\sigma_w^2 = \sum_{m,n}\mathbf{h}(m)\mathbf{R}_{\boldsymbol{\eta}\boldsymbol{\eta}}(n-m)\mathbf{h}^\dagger(n) = \mathbf{h}\mathbf{R}_{\boldsymbol{\eta}}\mathbf{h}^\dagger \qquad (9)$$

Using (9) and (6) in (8) yields the following.

$$J_{\text{noise}} = \frac{\mathbf{v}^\dagger\left[\frac{1}{\sigma_x^2}(\mathbf{U}^{-1})^\dagger\mathbf{R}_{\boldsymbol{\eta}}(\mathbf{U}^{-1})\right]\mathbf{v}}{\mathbf{v}^\dagger\mathbf{v}} = \frac{\mathbf{v}^\dagger\mathbf{Q}\mathbf{v}}{\mathbf{v}^\dagger\mathbf{v}} \qquad (10)$$

where $\mathbf{Q} \triangleq \frac{1}{\sigma_x^2}(\mathbf{U}^{-1})^\dagger\mathbf{R}_{\boldsymbol{\eta}}(\mathbf{U}^{-1})$. Since $\mathbf{R}_{\boldsymbol{\eta}}$ is Hermitian (as $\mathbf{R}^\dagger_{\boldsymbol{\eta}\boldsymbol{\eta}}(-k) = \mathbf{R}_{\boldsymbol{\eta}\boldsymbol{\eta}}(k)$), it follows that $\mathbf{Q}$ is also Hermitian. Thus, we have expressed J_{noise} as a Rayleigh quotient in terms of the vector $\mathbf{v}$. Combining (7) and (10), we obtain from (1),

$$J = \frac{\mathbf{v}^\dagger\left[\alpha\mathbf{P} + (1-\alpha)\mathbf{Q}\right]\mathbf{v}}{\mathbf{v}^\dagger\mathbf{v}} = \frac{\mathbf{v}^\dagger\mathbf{T}\mathbf{v}}{\mathbf{v}^\dagger\mathbf{v}}$$

where $\mathbf{T} \triangleq \alpha\mathbf{P} + (1-\alpha)\mathbf{Q}$. Since α is real and $\mathbf{P}$ and $\mathbf{Q}$ are Hermitian, it follows that $\mathbf{T}$ is also Hermitian. As such, it follows by *Rayleigh's principle* [5] that as $\mathbf{v}$ varies over all nonzero vectors, the minimum value of the objective function J is λ_{min}, where λ_{min} denotes the smallest eigenvalue of $\mathbf{T}$. This minimum value is achieved if $\mathbf{v} = \mathbf{v}_{\text{min}}$, where $\mathbf{v}_{\text{min}}$ denotes an eigenvector of $\mathbf{T}$ corresponding to λ_{min}. (More generally, the minimum value of J is achieved iff $\mathbf{v}$ is in the eigenspace corresponding to λ_{min}. However, for sake of clarity, we will ignore this scenario.) Hence, if J_{opt} and $\mathbf{h}_{\text{opt}}$ denote the optimum value of the objective function J and optimizing equalizer coefficients, respectively, then we have,

$$\boxed{\begin{aligned} J_{\text{opt}} &= \lambda_{\text{min}} \\ \mathbf{h}_{\text{opt}} &= \mathbf{v}^\dagger_{\text{min}}(\mathbf{U}^{-1})^\dagger \end{aligned}}$$

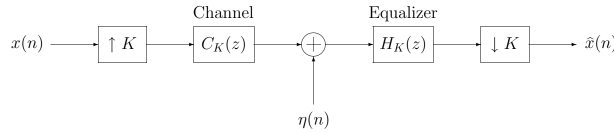

Fig. 2. Discrete-time model of a K-fold FSE.

The vector $\mathbf{h}_{\text{opt}}$ is referred to as an *eigenfilter* [9, 6] as its elements are filter coefficients derived from an eigenvector of a matrix. We now proceed to show that the FSE is simply a special case of the structure in Figure 1.

IV. Relation to Fractionally Spaced Equalizers

The discrete-time model of a K-fold FSE is shown in Figure 2 [8, 10]. Here, $C_K(z)$ and $H_K(z)$ represent, respectively, a K-fold oversampled version of our original channel and equalizer. The noise process $\eta(n)$ is similarly a K-fold oversampled version of our original noise process. Consider the following polyphase decompositions [11] of $C_K(z)$ and $H_K(z)$ below.

$$\begin{aligned} C_K(z) &= \sum_{k=0}^{K-1} z^k R_k(z^K) \qquad \text{(Type II)} \\ H_K(z) &= \sum_{k=0}^{K-1} z^{-k} E_k(z^K) \qquad \text{(Type I)} \end{aligned}$$

Using the *noble identities* [11], the structure in Figure 2 can be redrawn as in Figure 1 where we have,

$$\begin{aligned} [\mathbf{C}(z)]_{k,0} &= R_k(z) \\ [\mathbf{H}(z)]_{0,k} &= E_k(z) \\ [\boldsymbol{\eta}(n)]_{k,0} &= \eta(Kn-k) \end{aligned}$$

for $0 \le k \le K-1$. As such, if our goal is to choose the coefficients of the equalizer $H_K(z)$ to jointly shorten the overall channel and minimize the noise-to-signal ratio, then they can be found using the eigenfilter approach of the previous section.

V. Experimental Results

We now proceed to analyze how our design method compares with other known methods. One important figure of merit used to measure the performance of a TEQ in a DMT system, such as the one shown in Figure 3, is the maximum achievable bit rate. In order to achieve the maximum possible bit rate, bits are allocated to different components of $\widehat{\mathbf{s}}(n)$ depending upon the strength of the signal with respect to the noise and intersymbol interference (ISI). Assuming that the input noise is Gaussian and that the components of $\widehat{\mathbf{s}}(n)$ are spectrally isolated and sufficiently of narrow bandwidth, which is only approximately true here, the subchannels of $\widehat{\mathbf{s}}(n)$ can be viewed as independent parallel Gaussian channels [7]. In this case, the number of bits per real dimension to allocate in the k-th component of

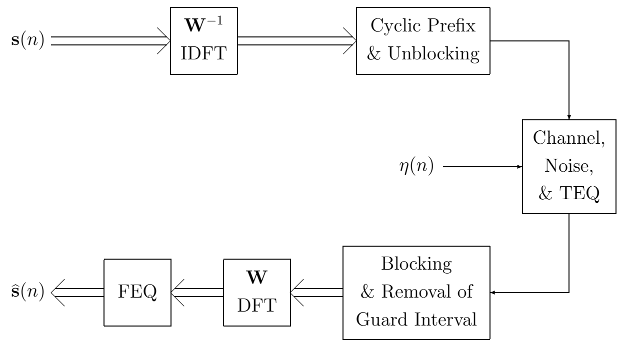

Fig. 3. Traditional DMT system.

$\widehat{\mathbf{s}}(n)$, denoted here by b_k, is simply the following [7].

$$b_k = \frac{1}{2}\log_2\left(1 + \frac{\text{SNR}_k}{\Gamma}\right)$$

Here, Γ is a "gap" quantity that depends on the coding and modulation format used as well as the desired probability of error. (For uncoded PAM and QAM constellations, $\Gamma = 9.8$ dB for a symbol error probability of 10^{-7} [7].)

To test our TEQ design method in a practical setting, such as the downstream link of an asymmetric digital subscriber line (ADSL) system, we have made the following assumptions.

- Input signal $\mathbf{s}(n)$ consists of QAM symbols.
- Desired probability of error is 10^{-7}.
- Discrete Fourier Transform (DFT) size is $N_{\text{DFT}} = 512$.
- Length of cyclic prefix is $N_{\text{CP}} = 32$.

As the input consists of two-dimensional QAM symbols, the number of bits to allocate in the k-th subchannel of $\widehat{\mathbf{s}}(n)$ is,

$$b_k = \left\lfloor \log_2\left(1 + \frac{\text{SNR}_k}{\Gamma}\right)\right\rfloor,\ 0 \le k \le N_{\text{DFT}} - 1$$

with $\Gamma = 9.8$ dB here. Under the approximate assumption that the subchannels are mutually isolated from each other and sufficiently narrowband, we have, from [2],

$$\text{SNR}_k = \frac{\sigma_x^2 |C_{\text{des}}(e^{j\omega_k})|^2}{\sigma_x^2 |C_{\text{res}}(e^{j\omega_k})|^2 + S_{ww}(e^{j\omega_k})},\ \omega_k = \frac{2\pi k}{N_{\text{DFT}}}$$

for $0 \le k \le N_{\text{DFT}} - 1$. Here $C_{\text{des}}(z)$ denotes the *desired* shortened channel response, i.e. a window of the $N_{\text{CP}} + 1$ most significant samples of $c_{\text{eff}}(n)$, and $C_{\text{res}}(z)$ denotes the *residual* channel response given by $C_{\text{res}}(z) = C_{\text{eff}}(z) - C_{\text{des}}(z)$. The numerator term corresponds to the observed signal power (before the FEQ), while the denominator terms correspond, respectively, to the ISI and noise powers (again before the FEQ).

Data for the channel and noise was obtained from the `Matlab DMTTEQ Toolbox` developed by G. Arslan, B. Lu, and B. L. Evans [3]. We used the following parameters here.

- Input power is $\sigma_x^2 = 14$ dBm.
- Length of equalizer is $L_e = 16$.

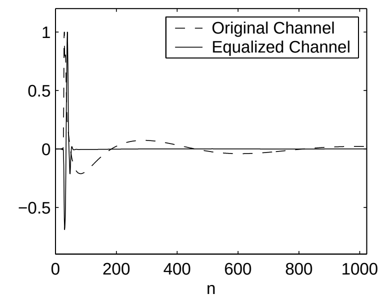

Fig. 4. Original and equalized channel impulse responses ($\alpha = 0.0028$, $n_{\text{mid}} = 39$, $f(n) = |n|$).

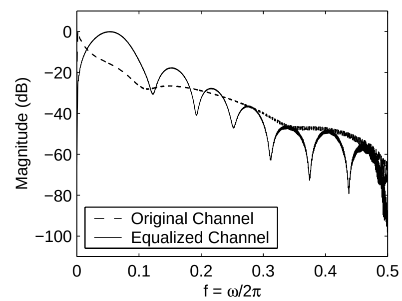

Fig. 5. Original and equalized channel magnitude responses ($\alpha = 0.0028$, $n_{\text{mid}} = 39$, $f(n) = |n|$).

- Carrier Service Area (CSA) loop # 1 was the channel.
- Input noise consists of near-end crosstalk (NEXT) noise plus additive white noise with power density -110 dBm/Hz.
- Sampling frequency is $f_s = 2.208$ MHz.

From this, the bit rate R_b was calculated using,

$$R_b = \frac{f_s}{(N_{\text{DFT}} + N_{\text{CP}})} \sum_{k=0}^{N_{\text{DFT}}-1} b_k$$

We varied the tradeoff parameter α and desired midpoint n_{mid}, as well as the penalty function $f(n)$, in order to obtain the greatest possible rate R_b. The best SSE that we obtained was for $\alpha = 0.0028$, $n_{\text{mid}} = 39$, and $f(n) = |n|$. Plots of the original and equalized channel impulse responses and magnitude responses are shown in Figures 4 and 5, respectively.

In addition to testing our proposed TEQ design method, we also considered the following methods.

- Delay spread minimization method of Schur and Speidel [6] (Special case of our method with $K = 1$, $\alpha = 1$, and $f(n) = n^2$).
- Eigenapproach of Farhang-Boroujeny and Ding [4].
- Geometric SNR maximization method of Al-Dhahir and Cioffi [1].

A plot of b_k as a function of the subcarrier index k for our equalizer is shown in Figure 6. Note that only the subcarrier indices $k = 0, \ldots, N_{\text{DFT}}/2$ are shown due to the mirror symmetry inherent in b_k due to the fact that the channel, equalizer,

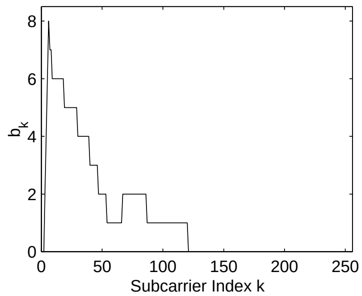

Fig. 6. Plot of b_k versus k using proposed method ($\alpha = 0.0028$, $n_{\text{mid}} = 39$, $f(n) = |n|$).

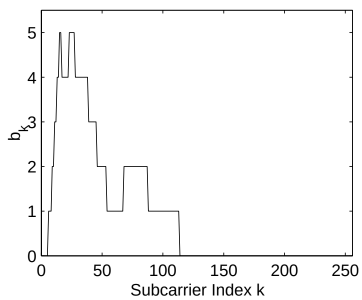

Fig. 7. Plot of b_k versus k using the method of [4].

and noise are all real here. In contrast to this, in Figure 7 we have plotted b_k versus k for the equalizer designed using the method of [4]. Although both methods do not allocate any bits for higher frequencies (near $\omega = \pi$), our method is able to allocate more bits for lower frequencies than the method of [4].

The observed bit rates for the various TEQs considered here are shown in Table 1. From the table, we can see that the SSE designed using our method yielded a higher bit rate than each of the other SSEs considered. More impressive, however, was the fact that the FSE TEQ designed using the proposed method yielded a much higher bit rate than any of the TEQs considered here. As before, α, n_{mid}, and $f(n)$ were chosen to yield the highest rate. It should be noted that the oversampled channel and noise shaping filters were created using the `interp1` command in `Matlab`. This example helps to justify the use of FSEs as TEQs for DMT systems. This improved performance is, of course, offset by the fact that the equalizer must operate at twice the rate of a normal TEQ.

VI. Concluding Remarks

We have shown that in terms of achievable bit rate, the TEQs designed using our approach were superior to those of other traditional methods. In particular, it was seen that of all of the methods considered, the FSE designed here performed the best. This helps to justify the future use of FSEs as TEQs in DMT systems. Use of a modified quadratic objective function and different choices for the penalty function $f(n)$ to further increase the bit rate are the subject of ongoing research.

Method	R_b (Mb/s)
Eigenfilter Method - SSE ($\alpha = 0.0028, n_{\text{mid}} = 39, f(n) = \|n\|$)	2.525
Schur & Speidel [6] ($n_{\text{mid}} = 42$)	2.176
Farhang-Boroujeny & Ding [4]	1.956
Al-Dhahir & Cioffi [1]	1.729
Eigenfilter Method - FSE ($K = 2, L_e = 8, \alpha = 0.998,$ $n_{\text{mid}} = 36, f(n)$ as in (4))	5.236

Table 1. Observed bit rates (R_b) for various equalizer methods.

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