A Generalized Deterministic Algorithm for Blind Channel Identification with Filter Bank Precoders

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***Abstract*— It is well-known that redundant filter bank precoders can be used for blind identification as well as equalization of FIR channels. Several algorithms have been proposed in the literature exploiting trailing zeros in the transmitter. In this paper we propose a generalized algorithm of which the previous algorithms are special cases. By carefully choosing system parameters, we can jointly optimize the system performance and computational complexity. Simulation shows that the proposed algorithm outperforms the previous ones when the parameters are optimally chosen.** [1]

I. Introduction

Wireless communication systems often suffer from a problem due to multi-path fading created by frequency-selective channels. Channel coefficients are often unknown to the receiver so that channel identification needs to be done before equalization can be performed. Among techniques for identifying unknown channel coefficients, blind methods have long been of great interest. In the literature many blind methods have been proposed based on the knowledge of second order statistics (SOS) or higher-order statistics of the transmitted symbols [6],[7]. These methods often need to accumulate a large number of received symbols until channel coefficients can be estimated accurately. This requirement leads to a disadvantage when the system is working over a fast-varying channel.

A deterministic blind method using redundant filterbank precoders was first proposed by Scaglione et al.[1] by exploiting trailing zeros introduced at the transmitter. Figure 1 shows a typical linearly redundant precoded system. Source symbols are divided into blocks with size M and linearly precoded into P-symbol blocks which are then transmitted to the channel. It is well known when $P \geq M + L$, where L is the maximum order of the FIR channel, inter-block interference (IBI) can be completely eliminated in the absence of noise. When the block size M increases, the bandwidth efficiency $\eta = (M + L)/M$ approaches unity asymptotically. The deterministic method proposed in [1] (which we will call SGB method) exploits trailing zeros with length L introduced in each transmitted block and assumes the input sequence is *rich*. That is, the matrix composed of finite source blocks achieves full rank.

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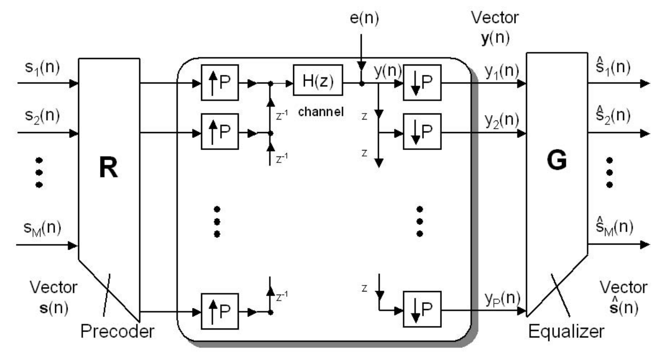

Fig. 1. Communication System with Redundant Filter Bank Precoders

The method in [1] requires the receiver to accumulate at least M blocks before channel coefficients can be identified. This still prevents the system from identifying channel coefficients accurately when the channel is fast-varying, especially when the block size M is large. More recently, Manton et al. pointed out that the channel could be identifiable with only two received blocks [2]. An algorithm based on viewing the channel identification problem as finding the greatest common divisor (GCD) of two polynomials is proposed in [3]. While greatly reducing the number of received blocks needed for channel identification, the algorithm has much more computational complexity especially when the block size M is large.

In this paper, we propose a generalized algorithm of which the SGB algorithm proposed in [1] and the GCD algorithm in [3] are both special cases. By carefully choosing parameters, the system performance and computational complexity can be jointly optimized. The rest of the paper is organized as follows. Section II describes the system structure with linear precoder filter banks and reviews the existing blind algorithms. In section III we present the generalized algorithm, derive the conditions on the input sequence under which the algorithm operates properly, and analyze the computational complexity with different system parameters. Simulation results are shown in section IV and conclusions are made in section V.

A. Notations

Boldfaced lower case letters represent column vectors, and boldfaced upper case letters are reserved for matrices. Super-

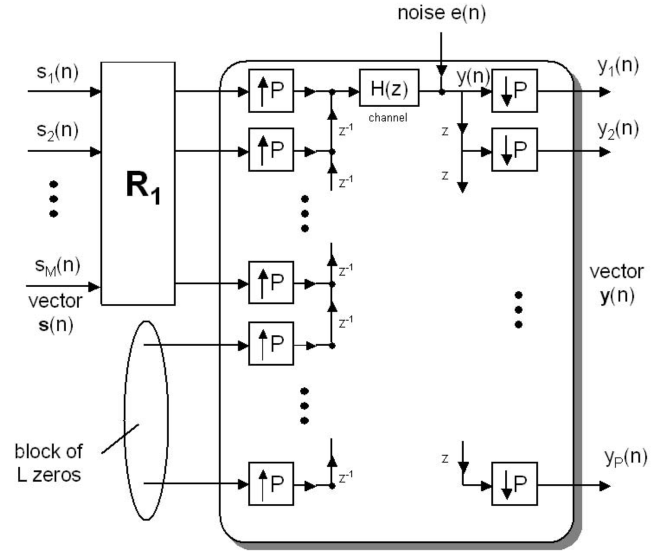

Fig. 2. The zero-padding system with precoder $\mathbf{R}_1$

scripts as in $\mathbf{A}^T$ and $\mathbf{A}^H$ denote the transpose and transpose-conjugate operations, respectively, of a matrix or a vector. All the vectors and matrices in this context are complex-valued.

If $\mathbf{v} = \begin{bmatrix} v_1 & v_2 & \cdots & v_M \end{bmatrix}^T$ is an $M \times 1$ column vector, then $\mathcal{T}(\mathbf{v}, q)$ denotes an $(M+q-1) \times q$ Toeplitz matrix whose first row and first column are $\begin{bmatrix} v_1 & 0 & \cdots & 0 \end{bmatrix}$ and $\begin{bmatrix} v_1 & v_2 & \cdots & v_M & 0 & \cdots & 0 \end{bmatrix}^T$, respectively.

II. Problem Formulation and Existing Results

A. Redundant Filter Bank Precoders

Consider a communication system depicted in Fig. 1. Source symbols $s_1(n), s_2(n), ..., s_M(n)$ may come from M different users or from a serial-to-parallel operation on data of a single user. For convenience we consider the blocked version $\mathbf{s}(n)$ as indicated. The vector $\mathbf{s}(n)$ is precoded by a $P \times M$ matrix $\mathbf{R}$ where $P > M$. The information with redundancy is then sent to the channel $H(z)$. We assume $H(z)$ is an FIR channel with a maximum order L, i.e., $H(z) = \sum_{k=0}^{L} h_k z^{-k}$. The signal is corrupted by a random noise $e(n)$ and then the received symbols $y(n)$ are further divided into $P \times 1$ block vectors $\mathbf{y}(n)$. The $M \times P$ matrix $\mathbf{G}$ is the channel equalizer and $\hat{s}_1(n), \hat{s}_2(n), ..., \hat{s}_M(n)$ are the recovered symbol streams. Also, for simplicity we define $\mathbf{h}$ as the column vector $\begin{bmatrix} h_0 & h_1 & \cdots & h_L \end{bmatrix}^T$. We set $P = M + L$, that is, the redundancy introduced in a block is equal to the maximum channel order.

B. Trailing Zeros as Transmitter Guard Interval

Throughout the paper we assume the precoder $\mathbf{R} = \begin{bmatrix} \mathbf{R}_1 \\ \mathbf{0} \end{bmatrix}$ where $\mathbf{R}_1$ is an $M \times M$ invertible matrix and the $L \times M$ zero matrix $\mathbf{0}$ represents zero-padding with length L in each transmitted block, as indicated in Fig. 2. In absence of noise, the received blocks can be written as

$$\underbrace{\begin{bmatrix} \mathbf{y}(1) & \mathbf{y}(2) & \cdots & \mathbf{y}(J) \end{bmatrix}}_{\mathbf{Y} \text{ matrix; size } P \times J} = \mathbf{H}\mathbf{R}_1 \underbrace{\begin{bmatrix} \mathbf{s}(1) & \mathbf{s}(2) & \cdots & \mathbf{s}(J) \end{bmatrix}}_{\mathbf{S} \text{ matrix; size } M \times J}$$

where $\mathbf{H} = \mathcal{T}(\mathbf{h}, M)$ is the Toeplitz channel matrix. As long as vector $\mathbf{h}$ is nonzero, the matrix $\mathbf{H}$ has full column rank M. Now, we assume the signal $\mathbf{s}(n)$ is *rich*, that is, there exists an integer J such that the matrix $\mathbf{S}$ has full row rank M. Since $\mathbf{R}_1$ is an $M \times M$ invertible matrix, we conclude that the $P \times J$ matrix $\mathbf{Y}$ has rank M. So there exists a $P \times L$ matrix $\mathbf{U}_0$ with L linearly independent columns which are annihilators of $\mathbf{Y}$, that is, $\mathbf{U}_0^H \mathbf{Y} = \mathbf{U}_0^H \mathbf{H}\mathbf{R}_1 \mathbf{S} = \mathbf{0}$. Now that $\mathbf{R}_1 \mathbf{S}$ has rank M, this implies

$$\mathbf{U}_0^H \mathbf{H} = \mathbf{U}_0^H \mathcal{T}(\mathbf{h}, M) = \mathbf{0}. \tag{1}$$

The channel coefficients $\mathbf{h}$ can then be determined by solving Eq. (1) in the least square sense. In practice where *channel noise* is present, the computation of the annihilators is replaced with the computation of the eigenvectors corresponding to the smallest L singular values of $\mathbf{Y}$. In this and the following sections, the channel noise term is not shown explicitly.

Note that this algorithm [1] works under the assumption that $\mathbf{S}$ has full row rank M. Obviously $J \geq M$ is a necessary condition for this assumption. This means the receiver must accumulate at least M blocks (i.e., a duration of $M(M+L)$ symbols) before channel identification can be performed. This could be a major disadvantage when the system is working over a fast-varying channel.

C. The GCD approach

Another approach proposed in [3] requires only two received blocks for blind channel identification. Recall that the channel is described by $\mathbf{y} = \mathbf{H}\mathbf{u} = \mathcal{T}(\mathbf{h}, M)\mathbf{u}$, or

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_P \end{bmatrix} = \begin{bmatrix} h_0 & & \mathbf{0} \\ h_1 & \ddots & \\ \vdots & & h_0 \\ h_L & & h_1 \\ & \ddots & \vdots \\ \mathbf{0} & & h_L \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_M \end{bmatrix}. \tag{2}$$

By multiplying $\begin{bmatrix} 1 & x & x^2 & \cdots & x^{P-1} \end{bmatrix}$ to both sides of Eq. (2), we obtain $y(x) = h(x)u(x)$, where $y(x) \triangleq y_1 + y_2 x + \cdots + y_P x^{P-1}$, $h(x) \triangleq h_0 + h_1 x + \cdots + h_L x^L$, and $u(x) \triangleq u_1 + u_2 x + \cdots + u_M x^{M-1}$ are polynomial representations of the output vector, channel vector, and input vector, respectively. This means, Eq. (2) is nothing but a polynomial multiplication. Now, suppose we have two received blocks $\mathbf{y}(1)$ and $\mathbf{y}(2)$, and let $y_1(x) = h(x)u_1(x)$ and $y_2(x) = h(x)u_2(x)$ represent the polynomial forms of these. Then the channel polynomial $h(x)$ can be found as the GCD of $y_1(x)$ and $y_2(x)$, given that the input polynomials $u_1(x)$ and $u_2(x)$ are co-prime to each other.

To compute the GCD of $y_1(x)$ and $y_2(x)$, we first construct a $(2P-1)\times 2P$ matrix

$$\mathbf{Y} \triangleq \begin{bmatrix} y_{11} & 0 & \cdots & 0 & y_{21} & 0 & \cdots & 0 \\ y_{12} & y_{11} & \ddots & \vdots & y_{22} & y_{21} & \ddots & \vdots \\ \vdots & y_{12} & \ddots & 0 & \vdots & y_{22} & \ddots & 0 \\ y_{1P} & \vdots & & y_{11} & y_{2P} & \vdots & & y_{21} \\ 0 & y_{1P} & & y_{12} & 0 & y_{2P} & & y_{22} \\ \vdots & \ddots & \ddots & \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & y_{1P} & 0 & \cdots & 0 & y_{2P} \end{bmatrix}.$$

One can verify that

$$\mathbf{Y} = \underbrace{\begin{bmatrix} h_0 & & \mathbf{0} \\ h_1 & \ddots & \\ \vdots & & h_0 \\ h_L & & h_1 \\ & \ddots & \vdots \\ \mathbf{0} & & h_L \end{bmatrix}}_{\substack{\text{matrix } \mathcal{H} \\ \text{size } (2P-1)\times(M+P-1)}} \underbrace{\begin{bmatrix} u_{11} & & \mathbf{0} & u_{21} & & \mathbf{0} \\ u_{12} & \ddots & & u_{22} & \ddots & \\ \vdots & & u_{11} & \vdots & & u_{21} \\ u_{1M} & & u_{12} & u_{2M} & & u_{22} \\ & \ddots & \vdots & & \ddots & \vdots \\ \mathbf{0} & & u_{1M} & \mathbf{0} & & u_{2M} \end{bmatrix}}_{\substack{\text{matrix } \mathcal{U} \\ ;\,\text{size } (M+P-1)\times 2P}}.$$

When $u_1(x)$ and $u_2(x)$ are co-prime to each other, the matrix $\mathcal{U}$ has full row rank $M+P-1$. Since $\mathcal{H}$ also has rank $M+P-1$, rank$(\mathbf{Y}) = M+P-1$ and hence $\mathbf{Y}$ has L left annihilators (i.e., there exists a $(2P-1)\times L$ matrix $\mathbf{U}_0$ such that $\mathbf{U}_0^H\mathbf{Y} = \mathbf{0}$). These annihilators are also annihilators of each column of matrix $\mathcal{H}$, and we can therefore identify channel coefficients $h_0, h_1, ..., h_L$ up to a scalar ambiguity.

III. A Generalized Algorithm

Comparing the two algorithms described in the previous section, we find that the GCD approach needs much fewer received symbols for blind identifiability. However, it has more computational complexity. Each received block is repeated P times to build a big matrix. In this section we will show that the number of repetitions of the blocks can be greatly reduced to save computation without degrading system performance significantly.

A. Algorithm Description

Observe Eq. (2) again and note that it is equivalent to

$$\mathcal{T}(y,Q) = \mathcal{T}(h, M+Q-1)\mathcal{T}(u,Q),$$

where Q can be any positive integer. Suppose the receiver gathers J blocks with $J \geq 2$. Then we have $\mathbf{Y}_Q^{(J)} = \mathbf{H}\mathbf{U}_Q^{(J)}$, where

$$\mathbf{Y}_Q^{(J)} = \left[\ \mathcal{T}(\mathbf{y}(1),Q) \quad \mathcal{T}(\mathbf{y}(2),Q) \quad \cdots \quad \mathcal{T}(\mathbf{y}(J),Q)\ \right],$$
$$\mathbf{H} = \mathcal{T}(h, M+Q-1),$$

and

$$\mathbf{U}_Q^{(J)} = \left[\ \mathcal{T}(\mathbf{R}_1\mathbf{s}(1),P) \quad \cdots \quad \mathcal{T}(\mathbf{R}_1\mathbf{s}(J),P)\ \right]. \tag{3}$$

Note that $\mathbf{U}_Q^{(J)}$ has size $(M+Q-1)\times QJ$ and $\mathbf{Y}_Q^{(J)}$ has size $(P+Q-1)\times QJ$. Assume now the matrix $\mathbf{U}_Q^{(J)}$ has full row rank $M+Q-1$. Taking singular-value decomposition (SVD) of $\mathbf{Y}_Q^{(J)}$ we have

$$\mathbf{Y}_Q^{(J)} = \left[\ \mathbf{U}_r\mathbf{U}_0\ \right]\begin{bmatrix} \mathbf{\Sigma} & \\ & \mathbf{0} \end{bmatrix}\left[\ \mathbf{V}_r\mathbf{V}_0\ \right]^H.$$

The size of $\mathbf{\Sigma}$ is $(M+Q-1)\times(M+Q-1)$ since both $\mathbf{H}$ and $\mathbf{U}_Q^{(J)}$ have full rank $(M+Q-1)$. The columns of the $(M+Q-1)\times L$ matrix $\mathbf{U}_0$ are left annihilators of matrix $\mathbf{Y}^{(J)}$ and also of $\mathbf{H}$ since $\mathbf{U}^{(J)}$ has full row rank. Suppose $\mathbf{U}_0$ has the structure

$$\mathbf{U}_0^H = \begin{bmatrix} u_{11} & u_{12} & \cdots u_{1,P+Q-1} \\ u_{21} & u_{22} & \cdots u_{2,P+Q-1} \\ \vdots & & \vdots \\ u_{L1} & u_{L2} & \cdots u_{L,P+Q-1} \end{bmatrix},$$

then we have

$$\underbrace{\left[\ \mathcal{U}_1^H \quad \mathcal{U}_2^H \quad \cdots \quad \mathcal{U}_L^H\ \right]^H}_{\mathcal{U} \text{ matrix; size } L(M+Q-1)\times(L+1)} \mathbf{h} = \mathbf{0}$$

where

$$\mathcal{U}_k \triangleq \begin{bmatrix} u_{k1} & u_{k2} & \cdots & u_{k,L+1} \\ u_{k2} & u_{k3} & \cdots & u_{k,L+2} \\ \vdots & & \vdots & \\ u_{k,M+Q-1} & u_{k,M+Q} & \cdots & u_{k,P+Q-1} \end{bmatrix}$$

for all k, $1 \leq k \leq L$. Vector $\mathbf{h}$ can thus be identified up to a scalar ambiguity.

B. System Parameters

The blind channel identification algorithm described above uses two parameters: the number of received blocks J and the number of repetitions per block Q. The algorithm works for any J and Q as long as $\mathbf{U}_Q^{(J)}$ has full row rank $M+Q-1$. *This is the only constraint for choosing parameters J and Q.* Note that if we choose $Q=1$ and $J \geq M$, then the algorithm reduces to the *SGB algorithm* [1]. If we choose $Q=P$ and $J=2$, it becomes the *GCD algorithm* [3].

Since $\mathbf{U}_Q^{(J)}$ has size $(M+Q-1)\times QJ$, $\mathbf{U}_Q^{(J)}$ having full row rank implies $QJ \geq M+Q-1$, or

$$Q \geq \frac{M-1}{J-1}. \tag{4}$$

Also note that we cannot choose $J=1$ since $\mathbf{U}_Q^{(J)}$ can never have full rank unless the block size $M=1$. This coincides with the theory that two blocks are required for blind channel identification [2]. While inequality (4) is a necessary condition for $\mathbf{U}_Q^{(J)}$ to have full rank, it is not sufficient. Nevertheless, when inequality (4) is satisfied, the probability of $\mathbf{U}_Q^{(J)}$ having full rank is usually close to unity in practice, especially when a large symbol constellation is used. Thus,

$$Q = \left\lceil \frac{M-1}{J-1} \right\rceil$$

appears to be a selection that minimizes the computational cost given the number of received blocks J.

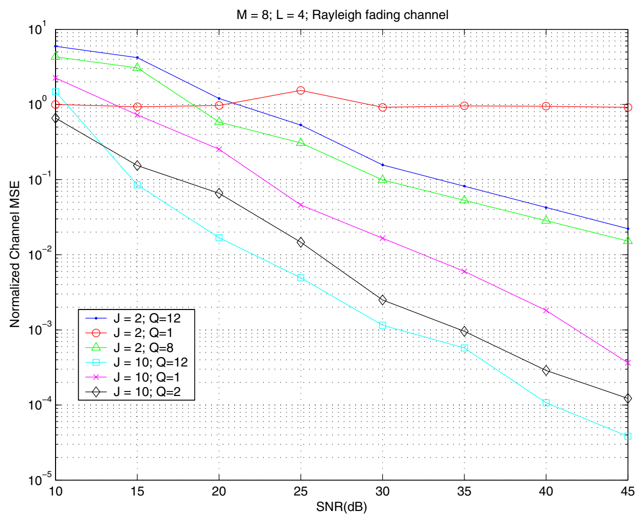

Fig. 3. Normalized least squared channel error estimation.

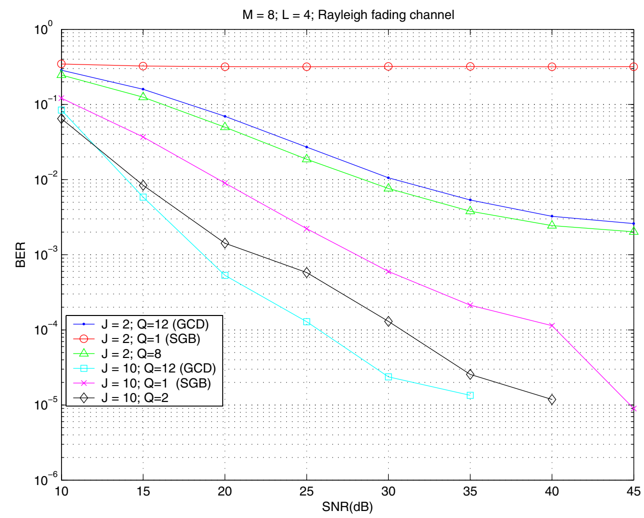

Fig. 4. Bit error rate.

When $J = 2$, Q can be chosen as small as $M - 1$ rather than P. If we take $J = 3$, $Q = \lceil \frac{M-1}{2} \rceil$ makes the matrix $\mathbf{Y}$ twice smaller. We can choose $Q = 1$ only when $J \geq M$. This coincides with the SGB algorithm which uses a richness assumption [1].

C. Complexity Analysis

The SVD computation dominates the complexity of the algorithm described above. The number of blocks J, the number of repetitions per block Q, and the received block size P decide the size of the matrix on which SVD is taken. The complexity of SVD operation on an $n \times m$ matrix [5] is on the order of $\mathcal{O}(mn^2)$ with $m \geq n$. Since $\mathbf{Y}_Q^{(J)}$ has size $(P + Q - 1) \times QJ$, the complexity is $\mathcal{O}(QJ(P + Q - 1)^2)$. We can see that the complexity can be greatly reduced by choosing a smaller Q. The SGB method [1] uses $Q = 1$ and the GCD method [3] uses $Q = P$. So the GCD method has a complexity around $4P$ times the complexity of the SGB method for any J. A choice of Q between 1 and P could be seen as a compromise between system performance and complexity. When J is large, we have the freedom to choose a smaller Q, as explained in the previous subsection.

IV. Simulations and Discussions

In this section, our proposed method is tested and compared with the existing methods [1], [3] described in Sec. II . A Rayleigh fading channel of order $L = 4$ is used. The size of transmitted blocks is $M = 8$ and received block size is $P = M + L = 12$. The normalized least squared channel error, denoted as E_{ch}, is used as the figure of merit for channel identification and is defined as follows.

$$E_{ch} = \frac{||\hat{\mathbf{h}} - \mathbf{h}||^2}{||\mathbf{h}||^2}$$

where $\hat{\mathbf{h}}$ and $\mathbf{h}$ are the estimated and the true channel vectors respectively. The simulated normalized channel estimation error is shown in Figure 3 and the corresponding BER is presented in Figure 4. When the number of blocks $J = 10$, the GCD method (with the number of block repetitions $Q = 12$) outperforms the SGB method ($Q = 1$) by a considerable range. Taking $Q = 2$ saves a lot of computation and yet still yields a good performance as indicated. In the case of $J = 2$, inequality (4) suggests that any system with $Q \geq 7$ works. Simulation results show that the system with $Q = 8$ even outperforms the original GCD method with $Q = 12$.

V. Concluding Remarks

In this paper we introduced a generalized algorithm for blind channel identification with linear redundant precoders. The number of accumulated received blocks can be chosen freely depending on the speed of channel variation. The minimum number of repetitions of each received block is derived to optimize the computation complexity while retaining good performance. Simulation shows that in certain cases the proposed algorithm outperforms existing methods.

In the future, finding the conditions on the input sequence and linear precoders so that the matrix $\mathbf{U}_Q^{(J)}$ defined in Eq. (3) has full rank remains an interesting theoretical question to answer. Also, extending this work to MIMO channels can be a challenging but important problem.

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