DISTRIBUTED ANTIPODAL PARAUNITARY PRECODERS FOR OFDM SYSTEMS

Borching Su and P. P. Vaidyanathan

Dept. of Electrical Engineering
California Institute of Technology, Pasadena, CA 91125, USA
E-mail: borching@systems.caltech.edu, ppvnath@systems.caltech.edu

ABSTRACT

Antipodal paraunitary (APU) matrices have been shown to be useful with respect to bit error rate (BER) as linear precoders for OFDM systems, especially in a fast-varying channel environment. In this paper we study a broader class of paraunitary matrices, namely, the distributed antipodal paraunitary (DAPU) matrices, of which APU matrices are special cases. Systematic methods for recursively generating certain types of DAPU matrices as well as a fast algorithm to implement them in a system are presented. Simulation results show that under the same distribution length, DAPU precoded systems with longer precoders have a better BER performance than those with APU precoders, especially in mid- and high-SNR region.[1]

1. INTRODUCTION

Linear precoded communication systems have been studied by a number of researchers[4-7]. In particular, antipodal paraunitary (APU) precoding technique for OFDM systems proposed by Phoong, Chang, and Lin [1] has been shown to have a much better bit error rate performance than the conventional OFDM system, especially when the channel is fast-varying, if an MMSE receiver is used. In this paper we are going to study a generalization of APU matrices, namely the *distributed antipodal paraunitary* (DAPU) matrices.

An $M\times M$ polynomial matrix $\mathbf{T}(z)$ is said to be *paraunitary*[2] (PU) if there exists a positive real number d such that $\tilde{\mathbf{T}}(z)\mathbf{T}(z) = d\mathbf{I}_M$. The tilde notation denotes $\tilde{\mathbf{T}}(z) = \mathbf{T}^H(1/z^*)$, where ^{H} is transpose-conjugation and * is the complex conjugation. If $d = 1$, $\mathbf{T}(z)$ is said to be *normalized paraunitary*. An $M \times M$ paraunitary matrix $\mathbf{T}(z) = \sum_{l=0}^{N-1} \mathbf{T}_l z^{-l}$ is called an *antipodal paraunitary* (APU) matrix[1] if its coefficient matrices $\mathbf{T}_l$ have entries restricted to be $\pm\frac{1}{\sqrt{MN}}$. APU matrices are also called lapped Hadamard matrices[3]. The factor $\frac{1}{\sqrt{MN}}$ ensures normalization. For convenience, we call a constant matrix $\mathbf{T}_l$ *antipodal* if all of its entries have the same magnitude, with either a positive or negative sign.

A polynomial matrix $\mathbf{T}(z)$ is said to be *distributed antipodal* (DA) if there exists a monotonically increasing nonnegative integer sequence $\{i_l\}_{l=0}^{N-1}$ such that

$$\mathbf{T}(z) = \sum_{l=0}^{N-1} \mathbf{T}_l z^{-i_l}, \tag{1}$$

where each $\mathbf{T}_l$ in (1) is antipodal. The integer sequence is called the *distribution vector* of the DA matrix. An $M \times M$ polynomial matrix is called a *distributed antipodal paraunitary*(DAPU) matrix if it is both DA and paraunitary.

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A precoded OFDM system with DAPU precoding matrix $\mathbf{T}(z)$ is shown in Fig.1. The block size of the system is equal to M, the size of $\mathbf{T}(z)$. The channel is characterized by a linear time-varying FIR system. We assume the channel remains constant during the period of transmitting an OFDM block and denote the impulse response of the channel as $C(n,k)$ when the kth block is transmitted. Assuming the receiver always knows the channel, a zero-forcing (ZF) receiving filter bank can be derived as $\mathbf{\Lambda}(k) = \mathbf{C}^{-1}(k)$, where $\mathbf{C}(k)$ is a diagonal matrix whose diagonal entries are samples of the Fourier Transform of the time-varying channel $C(n,k)$. Since $\mathbf{T}(z)$ is paraunitary, the system guarantees perfect reconstruction (PR) of symbols in absence of noise. In the presence of noise, we can further reduce the mean square error (MSE) of equalized symbols by applying an MMSE receiver [1]

$$\mathbf{\Lambda}(k) = E_s\mathbf{C}^H(k)\left(E_s\mathbf{C}(k)\mathbf{C}^H(k) + N_0\mathbf{I}_M\right)^{-1}, \tag{2}$$

where E_s is the expected value of symbol energy and N_0 is the noise variance.

In this paper, we first study the existence and construction of DAPU matrices (Sec. 2) and then use them in OFDM applications. In Sec. 3 we analyze the mean square error (MSE) of received symbol and bit error rate (BER) performance of the proposed system. Simulation results will be shown in Sec. 4 and a conclusion made in Sec. 5.

2. EXISTENCE AND GENERATION OF DAPU MATRICES

The existence of DAPU matrices depends on the size of the matrix M and the distribution vector $\{i_0, i_1, ..., i_{N-1}\}$. The length of this vector, N, is called the *distribution length*. In this section we are interested in how these factors (i.e., M, N, and $\{i_l\}_{l=0}^{N-1}$) affect the existence of DAPU matrices and how these matrices can be constructed.

For convenience, we also define the *distribution pattern* to be a string composed of 0's and 1's that represents the time-domain distribution of a DAPU matrix. For example, distribution vector $\{0, 1, 3, 4\}$ corresponds to distribution pattern [11011]. Observe that if the distribution vector is $\{0, 1, ..., N-1\}$ (i.e., the distribution pattern is [111...1]), then this DAPU matrix reduces to an APU matrix.

Existence conditions and several methods for generation of APU matrices have been studied in [3]. The APU matrices have the most "compact" distribution pattern among DAPU matrices that have the same distribution length N, since they do not have zero terms among N coefficient matrices. In this section, we will develop methods for generating DAPU matrices, partly by extend-

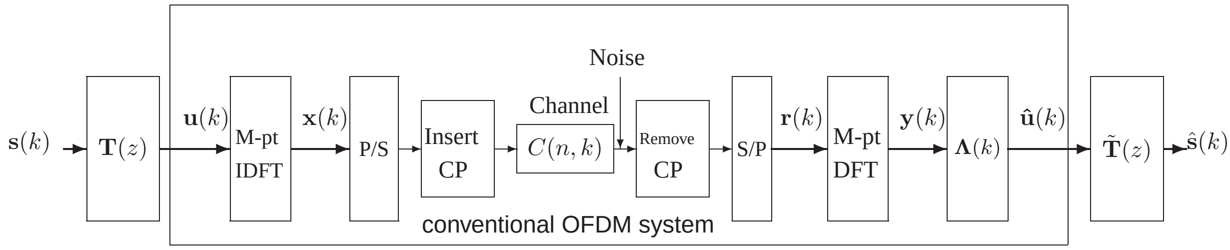

Fig. 1: An OFDM system with DAPU precoding matrix $\mathbf{T}(z)$, based on cyclic prefix(CP).

ing existing methods used for generating APU matrices. Furthermore, for distribution patterns that are "recursively symmetric" (to be defined), we have been able to find a fast implementation.

An obvious way to generate a DAPU matrix is simply interpolating an APU matrix by a factor J. If $\mathbf{T}(z)$ is an $M \times M$ APU matrix with length N, then $\mathbf{T}(z^J)$ is an $M \times M$ DAPU matrix with distribution vector $\{0, J, ..., (N-1)J\}$. More generalized construction methods will be presented next.

2.1. Recursive method

Lemma 1: Suppose $\mathbf{T}^{(k)}(z) = \begin{bmatrix}\mathbf{A}(z) & \mathbf{B}(z)\end{bmatrix}$ is an $M \times M$ normalized DAPU matrix where M is even and $\mathbf{A}(z)$ and $\mathbf{B}(z)$ are both $M \times \frac{M}{2}$ matrices. The distribution pattern of $\mathbf{T}^{(k)}(z)$ is $\{i_l\}_{l=0}^{N_k-1}$. The length of $\mathbf{T}^{(k)}(z)$ is $L_k = i_{N_k-1} + 1$. Then,

$$\mathbf{T}^{(k+1)}(z) = \frac{1}{\sqrt{2}} \begin{bmatrix}\mathbf{A}(z) + z^{-d_k}\mathbf{B}(z) & \mathbf{A}(z) - z^{-d_k}\mathbf{B}(z)\end{bmatrix}$$

is also a normalized DAPU matrix whose distribution vector is $\{i_l\}_{l=0}^{N_k-1} \bigcup \{i_l + d_k\}_{l=0}^{N_k-1}$if $d_k \geq L_k$. The distribution length N_{k+1} is $2N_k$, and the length of $\mathbf{T}^{k+1}(z)$ is $L_{k+1} = L_k + d_k$.

Proof: We first show that $\mathbf{T}^{(k+1)}(z)$ is distributed antipodal. Since $\mathbf{T}^{(k)}(z)$ is DA with distribution pattern $\{i_l\}_{l=0}^{N_k-1}$, so are $\mathbf{A}(z)$ and $\mathbf{B}(z)$. Since $d_k \geq L_k$ and thus $d_k > i_{N_k-1}$, there will be no overlapping between sets $\{i_l\}_{l=0}^{N_k-1}$ and $\{i_l + d_k\}_{l=0}^{N_k-1}$. Therefore, $\mathbf{A}(z) \pm z^{-d_k}\mathbf{B}(z)$, and hence $\mathbf{T}^{(k+1)}(z)$, constitute a longer DA matrix whose distribution pattern is $\{i_l\}_{l=0}^{N_k-1} \bigcup \{i_l + d_k\}_{l=0}^{N_k-1}$.

As for paraunitarity, since $\mathbf{T}^{(k)}(z)$ is normalized PU, we have $\tilde{\mathbf{A}}(z)\mathbf{A}(z) = \tilde{\mathbf{B}}(z)\mathbf{B}(z) = \mathbf{I}_{M/2}$ and $\tilde{\mathbf{A}}(z)\mathbf{B}(z) = \tilde{\mathbf{B}}(z)\mathbf{A}(z) = \mathbf{O}_{M/2}$. With these, one can easily verify that $\tilde{\mathbf{T}}^{(k+1)}(z)\mathbf{T}^{(k+1)}(z) = \mathbf{I}_M$. So $\mathbf{T}^{(k+1)}(z)$ is also normalized PU. □

Using Lemma 1, we can generate DAPU matrices with various distribution patterns recursively. Starting from an $M \times M$ normalized Hadamard matrix: $\mathbf{T}^{(0)}(z) = \mathbf{H}$, whose length is $L_0 = 1$, the choice of $\{d_i\}_{i=0}^{k-1}$ decides the generated pattern, where d_i must satisfy $d_i > \sum_{j=0}^{i-1} d_j, \forall i > 0$. Notice that if we choose $\{d_i\}_{i=0}^{k-1} = \{1, 2, 4, 8, ...\}$, then the matrices generated by this method reduce to APU matrices. Consider the set of all possible distribution patterns generated by this recursive method. We can show by induction that these patterns are always symmetric. Note that any pattern in this set whose length is longer than unity is composed by inserting some padding zeros between two identical shorter patterns in the same class. Since these two identical patterns are symmetric by induction, the newly generated pattern is also symmetric regardless of the number of padding zeros. We call this special kind of patterns *recursively symmetric.* Some possible recursively symmetric patterns are listed in Table 1.

N	$\{d_i\}$	Distribution vector$\{i_l\}$	Distribution pattern	Remark
1		{0}	[1]	Hadamard
2	1	{0,1}	[1 1]	APU(z)
2	2	{0,2}	[1 0 1]	APU(z^2)
2	3	{0,3}	[1 0 0 1]	APU(z^3)
2	4	{0,4}	[1 0 0 0 1]	APU(z^4)
4	1,2	{0,1,2,3}	[1 1 1 1]	APU(z)
4	1,3	{0,1,3,4}	[1 1 0 1 1]	
4	2,3	{0,2,3,5}	[1 0 1 1 0 1]	
4	1,4	{0,1,4,5}	[1 1 0 0 1 1]	
4	2,4	{0,2,4,6}	[1 0 1 0 1 0 1]	APU(z^2)
4	3,4	{0,3,4,7}	[1 0 0 1 1 0 0 1]	
4	1,5	{0,1,5,6}	[1 1 0 0 0 1 1]	
4	2,5	{0,2,5,7}	[1 0 1 0 0 1 0 1]	
4	3,5	{0,3,5,8}	[1 0 0 1 0 1 0 0 1]	
4	4,5	{0,4,5,9}	[1 0 0 0 1 1 0 0 0 1]	
4	3,6	{0,3,6,9}	[1 0 0 1 0 0 1 0 0 1]	APU(z^3)

Table 1: Recursively symmetric patterns for DAPU matrices.

2.2. Butterfly Method and Fast Algorithm

A different method for generating recursively symmetric DAPU matrices will be described. Let M be even and define two $M \times M$ matrices: $\mathbf{\Theta}(z) = \mathrm{diag}\begin{bmatrix} 1 & z^{-1} & 1 & z^{-1} & \cdots & 1 & z^{-1}\end{bmatrix}$ and $\mathcal{B}_M = \mathbf{I}_{M/2} \otimes \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$, where $\otimes$ denotes the Kronecker product, then we have the following lemma.

Lemma 2: Suppose $\mathbf{T}^{(k)}(z)$ is an $M \times M$ normalized DAPU matrix whose distribution vector is $\{i_l\}_{l=0}^{N_k-1}$ and whose length is L_k. Then

$$\mathbf{T}^{(k+1)}(z) = \frac{1}{\sqrt{2}} \mathcal{B}_M \mathbf{\Theta}(z^{d_k}) \mathbf{T}^{(k)}(z)$$

is also a normalized DAPU matrix whose distribution vector is $\{i_l\}_{l=0}^{N_k-1} \bigcup \{i_l + d_k\}_{l=0}^{N_k-1}$, if $d_k \geq L_k$. The distribution length N_{k+1} is $2N_k$, and the length of $\mathbf{T}^{k+1}(z)$ is $L_{k+1} = L_k + d_k$.

Proof: Notice that $\mathcal{B}_M \mathbf{\Theta}(z^{d_k})$ has the form $\mathbf{I}_{M/2} \otimes \begin{bmatrix} 1 & z^{-d_k} \\ 1 & -z^{-d_k} \end{bmatrix}$. When multiplied by a DA matrix $\mathbf{T}^{(k)}(z)$, it leads to distribution vectors $\{i_l\}_{l=0}^{N_k-1}$ and $\{i_l + d_k\}_{l=0}^{N_k-1}$. Since $d_k \geq L_k$, these two parts do not overlap and thus $\mathbf{T}^{(k+1)}(z)$ is also DA. Paraunitarity can be easily verified since $\mathcal{B}_M$, $\mathbf{\Theta}(z)$, and $\mathbf{T}^{(k)}(z)$ are all paraunitary. □

Lemma 2 provides us an alternative method, namely the *butterfly method*, to generate DAPU matrices with recursively symmetric distribution patterns, starting from a normalized $M \times M$ Hadamard matrix $\mathbf{T}^{(0)}(z) = \mathbf{H}$. The generated matrices may not be the same as those generated by Lemma 1. However, since they

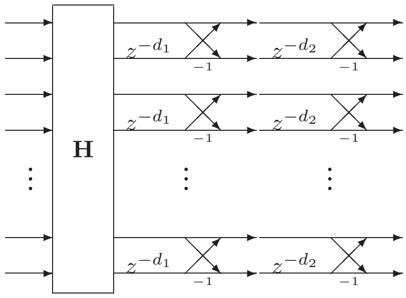

Fig. 2: Fast algorithm for butterfly method.

have the same distribution vectors, as we will show in Sec. 3, they have exactly the same performance when applied as precoders for OFDM systems. Furthermore, transformation of an $M \times M$ DAPU matrix generated by the butterfly method can be implemented with a fast algorithm, as shown in Fig. 2, in $M(\log M + \log N)$ additions per OFDM block.

2.3. DAPU Matrices with Arbitrary Patterns

Patterns of DAPU matrices generated by the two methods above are always recursively symmetric. DAPU matrices with arbitrary distribution patterns, however, can be generated by the following method, if the distribution length N equals the size of a Hadamard matrix.

Lemma 3: Let $\mathbf{H}$ be an $M \times M$ normalized Hadamard matrix, $N = M$, and $\{i_0, i_1, ..., i_{N-1}\}$ be arbitrarily specified, then

$$\mathbf{T}(z) = \mathbf{H}\,\mathrm{diag}\left[\begin{array}{cccc} z^{-i_0} & z^{-i_1} & \cdots & z^{-i_{N-1}} \end{array}\right]\mathbf{H}$$

is a normalized DAPU with distribution vector $\{i_0, i_1, ..., i_{N-1}\}$.$\square$

Using Lemma 3, we can generate DAPU matrices with arbitrary distribution patterns, with the only constraint that the distribution length N must be equal to the matrix size M. However, if there exists positive integer n such that $M = 2^n N$, we can generate an $N \times N$ DAPU matrix $\mathbf{T}^{(0)}(z)$ first and then expand the size of the matrix up to M using the recursion

$$\mathbf{T}^{(k+1)}(z) = \frac{1}{\sqrt{2}}\left[\begin{array}{cc} \mathbf{T}^{(k)}(z) & \mathbf{T}^{(k)}(z) \\ \mathbf{T}^{(k)}(z) & -\mathbf{T}^{(k)}(z) \end{array}\right], k = 0, 1, ..., n-1.$$

3. DAPU PRECODED OFDM SYSTEMS

The cyclic prefix eliminates the interblock interference (IBI) and the M-point FFT and IFFT blocks diagonalize the channel. As a result, an equivalent vectorized system representation of Fig. 1 is shown in Fig. 3, where $\mathbf{C}(k)$ is a diagonal matrix whose diagonal entries represent samples of the channel frequency response at the time the kth block is transmitted, and $\boldsymbol{\nu}(k)$ is a noise vector whose components are independently identically distributed random variables with variance N_0. The receiver matrix, $\boldsymbol{\Lambda}(k)$, can be chosen as the inverse of $\mathbf{C}(k)$ for a ZF receiver, or be defined as in (2) for an MMSE receiver. In either case $\boldsymbol{\Lambda}(k)$ is a diagonal matrix.

3.1. Noise Analysis without Precoders

We denote the ith diagonal entry of $\mathbf{v}(k)$ as $v_i(k)$, where $\mathbf{v}$ can be any bold-face lower case vector in the context. We will also denote the ith diagonal entries of $\boldsymbol{\Lambda}(k)$ and $\mathbf{C}(k)$ as $\lambda_i(k)$ and $C_i(k)$, respectively. Define the noise vector in the kth block $\boldsymbol{\beta}(k) =$

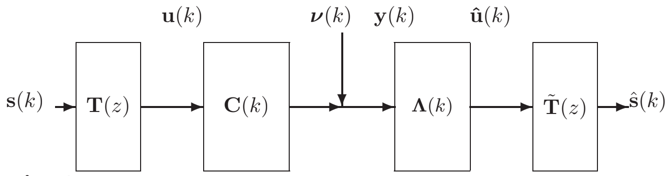

Fig. 3: Equivalent DAPU precoded-OFDM system based on cyclic prefix.

$\hat{\mathbf{u}}(k) - \mathbf{u}(k)$. The autocorrelation matrices of $\boldsymbol{\beta}(k)$ given by $\mathcal{R}_\beta(k,l) = E[\boldsymbol{\beta}(k)\boldsymbol{\beta}^H(k-l)] = \delta(l)\mathcal{R}_\beta(k,0)$ can be shown to be diagonal matrices[1]. It is also known that [1] the ith diagonal entry of $\mathcal{R}_\beta(k,0)$ is equal to the noise variance at the ith subchannel $E[|\beta_i(k)|^2]$, which would be $N_0/|C_i(k)|^2$ if a ZF receiver is used or $N_0/(|C_i(k)|^2 + N_0/E_s)$ if an MMSE receiver is used.

3.2. Averaging Effect on Noise Variance Introduced by DAPU Matrices

Defining the output noise vector $\mathbf{e}(k) = \hat{\mathbf{s}}(k) - \mathbf{s}(k)$, we have

$$\mathbf{e}(k) = \sum_{l=0}^{N-1} \mathbf{T}_{i_l}^H \boldsymbol{\beta}(k + i_l),$$

and the zeroth autocorrelation matrix of $\mathbf{e}(k)$ is given by

$$\mathcal{R}_e(k,0) = E[\mathbf{e}(k)\mathbf{e}^H(k)] = \sum_{l=0}^{N-1} \mathbf{T}_{i_l}^H \mathcal{R}_\beta(k + i_l, 0)\mathbf{T}_{i_l}$$

Using the fact that $\mathbf{T}_{i_l}$ is antipodal and $\mathcal{R}_\beta(k + i_l, 0)$ is diagonal, the output noise variance at the ith subchannel at time k is:

$$\begin{aligned} \sigma_{i,\mathbf{T}}^2(k) &= \text{the } i\text{th diagonal entry of } \mathcal{R}_{\mathbf{e}}(k,0) \\ &= \frac{1}{N}\sum_{l=0}^{N-1}\left[\frac{1}{M}\sum_{n=0}^{M-1} E[|\beta_i(k+i_l)|^2]\right]. \end{aligned}$$

Here $E[|\beta_i(k + i_l)|^2]$ depends on whether a ZF or an MMSE receiver is used. Its value can be found as in Sec 3.1.

Notice that the quantity $\sigma_{i,\mathbf{T}}^2(k)$ is independent of i: all subchannels have the same noise variance, which is the average of non-precoded noise variances among M subchannels and over the N blocks specified by the distribution vector $\{i_l\}_{l=0}^{N-1}$.

Just as what has been shown in [1], introducing a DAPU precoder $\mathbf{T}(z)$ into the system does not change the mean square error(MSE) of the detected symbol. It only redistributes the error. This also explains why the MMSE receiver of the precoded OFDM system is the MMSE receiver $\boldsymbol{\Lambda}(k)$ defined in (2) followed by $\tilde{\mathbf{T}}(z)$. (However, a more rigorous proof can be given by applying the orthogonality principle.)

We should also note that the averaging effect depends only on distribution vectors. Two different DAPU matrices with the same distribution vector have exactly the same averaging effect when applied as precoders for OFDM systems and hence have the same performance.

If we choose a longer DAPU precoder, the averaging effect involves noise coming from more distant blocks, where the channel state could differ more. Thus, a longer DAPU precoder could have more “channel diversity” than an APU precoder with the same distribution length N. As we will see in the next section, the performance of systems with longer DAPU precoders is better.

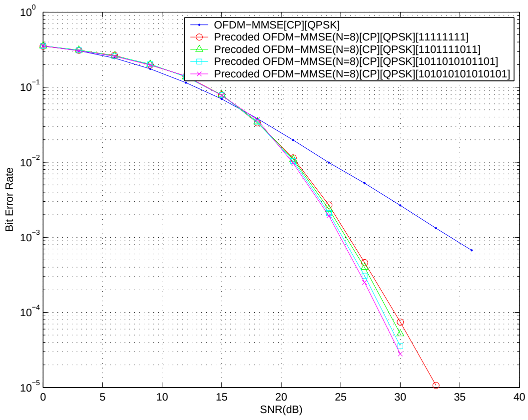

Fig. 4: BER performance comparison for conventional, APU precoded, and DAPU precoded OFDM systems with different distribution patterns in fast varying channels.

4. SIMULATION RESULTS

We carry out Monte-Carlo experiments to compare performances of conventional OFDM systems and precoded OFDM systems with precoders of different lengths. The time-variant FIR channel $C(n,k)$ is generated by linear interpolation described as follows: given a parameter called *varying interval* T, and maximum order of FIR channel L, the channel coefficients are defined as:

$$C(n,k) = \begin{cases} \mu(\frac{n}{T},k), \text{if } n \text{ is multiple of } T \\ \lambda\mu(\lfloor\frac{n}{T}\rfloor,k) + (1-\lambda)\mu(\lceil\frac{n}{T}\rceil,k), \text{otherwise} \end{cases}$$

$\forall n \geq 0, 0 \leq k \leq L$, where $\lambda = \lceil\frac{n}{T}\rceil - \frac{n}{T}$ and $\mu(t,k) \sim \mathcal{CN}(0,1)$ are i.i.d. complex Gaussian random variables $\forall t \geq 0, 0 \leq k \leq L$. The varying interval T decides how fast the channel is varying. A smaller T indicates the channel is changing faster.

In our simulation, we used $M = N = 8$, $L = 4$, and QPSK symbol modulation. The channel noise $\nu(n)$ is AWGN with variance N_0. We assume the receiver knows the exact channel response. Since using ZF receivers in precoded systems would cause even worse results than conventional OFDM systems[1], we use only MMSE receivers in our simulation. In order to assure simulation accuracy, each data point in the BER plots has at least accumulated 1,000 occurrences of errors before being shown.

The results for $T = 10$ are shown in Fig. 4. From the figure, we see that the BER performance of APU precoded OFDM system (with the most "compact" distribution pattern, [11111111]) is much better than conventional OFDM system in mid- and high-SNR region, (as argued in [1]). Under the same distribution length $N = 8$, DAPU matrices have even better performances, and a longer distribution pattern yield a better BER performance. If we compare cases [11111111] and [101010101010101], we can find that "stretching" the length of precoders two times can yield an additional gain of more than 1 dB when the bit error rate is 10^{-4}.

For a channel that is varying 10 times slower (i.e., $T = 100$), the results are shown in Fig. 5. In [1] it is argued that with a slowly-varying channel, APU precoders have less improvement from conventional OFDM systems than in fast-varying channel case. But from Fig. 5 we see the improvement by using longer DAPU matrices is even larger. An additional gain of more than 1.5dB is achieved at BER = 10^{-4} by replacing APU precoders with DAPU precoders with twice the length.

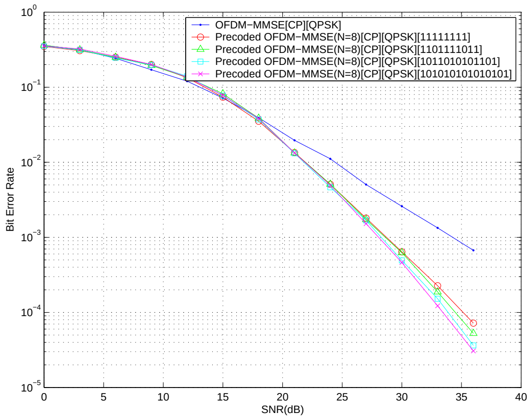

Fig. 5: BER performance comparison for conventional, APU precoded, and DAPU precoded OFDM systems with different distribution patterns in slowly varying channels.

5. CONCLUDING REMARKS

In this paper we studied distributed antipodal paraunitary (DAPU) matrices and used them as precoders for OFDM systems. Theoretical and simulation results show that under the same distribution length N, DAPU matrices yield a better BER performance than conventional APU precoders, especially when the channel does not vary too fast. This allows us to improve the performance of precoded OFDM systems without increasing computational complexity and peak-to-average power ratio. The only price paid for the use of DAPU is longer delay. We also presented several methods for finding DAPU matrices and a fast implementation method. Possible future works include finding optimal distribution patterns for specific channel characteristics and finding methods for generating wider classes of DAPU matrices.

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