On the Persistency of Excitation for Blind Channel Estimation in Cyclic Prefix Systems

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***Abstract*— Recently, a new subspace-based blind channel estimation algorithm in cyclic prefix (CP) system was reported. A persistency of excitation (PE) property of the input signal is required for the algorithm to work. In this paper, the probability of fulfilling the PE property under different situations is studied. Four factors in the algorithm affect the PE property of the input signal: 1) signal constellation used; 2) precoder coefficients; 3) number of consecutive blocks; 4) a number called the repetition index. Theoretical derivations as well as numerical simulations are given to demonstrate the main points of this paper. Important conclusions are 1) that the probability of fulfilling the PE property increases and converges to unity when the number of received blocks increases but is always upper-bounded by a value less than unity when the repetition index increases; 2) that the probability of fulfilling the PE property is smaller when the algorithm is applied in orthogonal frequency division multiplexing (OFDM) systems than in single-carrier-cyclic-prefix (SC-CP) systems.** [1]

I. Introduction

Blind estimation of channel coefficients for communication systems with cyclic prefix (CP) in the transmitter has recently become a problem of interest in the literature [1]–[5]. This is partly due to the growing popularity of CP systems in the standards of orthogonal frequency division multiplexing (OFDM) and single-carrier cyclic prefix (SC-CP) systems. Among many blind methods proposed in the literature, methods based on subspace decomposition [1]–[4] are considered to possess attractive features such as applicability to arbitrary signal constellations with a reasonable computational complexity. However, subspace-based methods usually rely on a certain kind of persistency of excitation (PE) property of the input signals. Under such a constraint, the minimum required number of received blocks (or received OFDM symbols) had been considered to be around twice the block size [4]. This becomes one of the disadvantages of a subspace-based blind estimation algorithm especially when the channel coefficients are fast-varying.

More recently, Su and Vaidyanathan proposed a new generalized subspace-based algorithms [11] using a concept of repetition index which requires a relaxed form of the PE property. The relaxation of the PE property makes the new algorithm potentially able to work using only *three* received blocks (or OFDM symbols). However, the reliability of the algorithm still depends on the conditions required to fulfill the PE property, which are still under investigation, especially when the number of blocks used at the receiver is small. In this paper, we study the conditions under which this PE criterion is satisfied. The roles of different constellations, precoders, number of received blocks, and repetition indices are considered. Several theorems will be derived and numerical simulations will also be presented to demonstrate the discoveries. An important result is that the PE property is more likely to be satisfied for SC-CP systems than OFDM systems.

The rest of this paper is organized as follows. Section II briefly reviews the algorithm proposed in [11]. In Section III, theories of persistency of excitation for the algorithm are presented. In Section IV, numerical simulations are presented to give a clearer view on the subject studied. Finally, conclusions are made in Section V.

A. Notations

Boldfaced lower case letters represent column vectors. Boldfaced upper case letters and calligraphic upper case letters are reserved for matrices. Superscripts ^{T}, and † as in $\mathbf{A}^T$, and $\mathbf{A}^\dagger$ denote the transpose and transpose-conjugate operations, respectively. All the vectors and matrices in this paper are complex-valued. The notation W_M denotes $e^{j2\pi/M}$, and $\mathbf{W}_M$ is the $M \times M$ normalized DFT matrix whose kl-th entry is $W_M^{-(k-1)(l-1)}/\sqrt{M}$. Column and row indices of all matrices and vectors begin at one. $\mathbf{I}_M$ is the $M \times M$ identity matrix, and $\mathbf{0}_{m\times n}$ is the $m \times n$ zero matrix. $\mathbb{C}$ is the set of all complex numbers. If $\mathbf{v} = \begin{bmatrix} v_1 & v_2 & \cdots & v_m \end{bmatrix}^T$ is an $m \times 1$ vector, we use $\mathcal{T}_n(\mathbf{v})$ to denote the $(m+n-1) \times n$ Toeplitz matrix [9] whose first column is $[\mathbf{v}^T, \mathbf{0}^T_{(n-1)\times 1}]^T$ and whose first row is $[v_1, \mathbf{0}_{1\times(n-1)}]$.

Due to the special property of cyclic prefixes, we will use the following notation extensively in this paper. Suppose $\mathbf{y}$ is an $m \times 1$ column vector $\mathbf{y} = \begin{bmatrix} y_1 & y_2 \cdots & y_m \end{bmatrix}^T$. Then the notation $[\mathbf{y}]_b^a$ denotes the $(b-a+1) \times 1$ vector

$$[\mathbf{y}]_b^a = \begin{bmatrix} y_a & y_{a+1} & \cdots & y_b \end{bmatrix}^T$$

if $1 \le a \le b \le m$. An extension of this definition to any arbitrary pair of integers a and b satisfying $a \le b$ is made by defining y_k as $y_{(k-1 \bmod m)+1}$ for any $k > m$ or $k < 1$. For example, if $\mathbf{y} = \begin{bmatrix} y_1 & y_2 & y_3 \end{bmatrix}^T$, then $[\mathbf{y}]_7^{-1}$ denotes the vector $\begin{bmatrix} y_2 & y_3 & y_1 & y_2 & y_3 & y_1 & y_2 & y_3 & y_1 \end{bmatrix}^T$.

[1]Work supported in parts by the NSF grant CCF-0428326, ONR grant N00014-06-1-0011, and the Moore Fellowship of the California Institute of Technology.

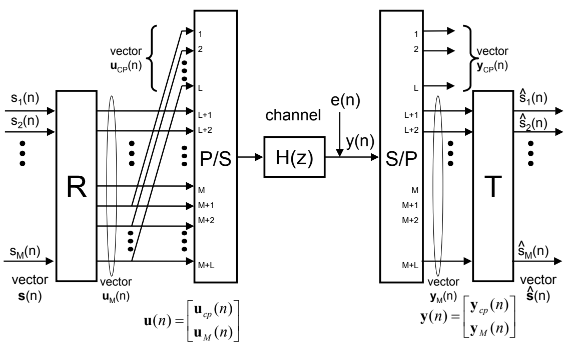

Fig. 1. A typical cyclic prefix system.

II. Review of the Blind Algorithm in CP Systems

Figure 1 shows a typical communication system with cyclic prefix (CP) introduced in each transmitted block. Source samples $s_k(n)$ are drawn from a finite signal constellation $\mathcal{S} \subset \mathbb{C} - \{0\}$. The signal $\mathbf{s}(n)$ passes through a linear precoder characterized as an $M \times M$ invertible matrix $\mathbf{R}$ and becomes $\mathbf{u}_M(n) = \mathbf{R}\mathbf{s}(n)$. A cyclic prefix $\mathbf{u}_{cp}(n) = [\mathbf{u}_M(n)]_M^{M-L+1}$ of length L is inserted at the beginning of each block and forms vectors $\mathbf{u}(n) = [\mathbf{u}_{cp}(n)^T, \mathbf{u}_M(n)^T]^T$ of size $P = M + L$. Vectors $\mathbf{u}(n)$ are then serialized and sent to the channel characterized as an FIR filter $H(z) = \sum_{k=0}^{L} h_k z^{-k}$ whose order is upper bounded by L. At the receiver each received sample is corrupted by an additive white Gaussian noise and then blocked into vectors $\mathbf{y}(n)$ of size P.

Given the number of collected blocks J and repetition index Q, the following algorithm blindly estimates the channel coefficients $\mathbf{h} = \begin{bmatrix} h_0 & h_1 & \cdots & h_L \end{bmatrix}$ using received blocks $\mathbf{y}(n)$ only, without knowledge of $\mathbf{u}(n)$. Due to space limit, the algorithm presented below is in its simplest form. Readers with interest in more details of the algorithm are referred to [10], [11].

Algorithm 1:

1) For two consecutive blocks $\mathbf{y}(n-1)$ and $\mathbf{y}(n)$, define

$$\bar{\mathbf{y}}_k(n) = \begin{bmatrix} [\mathbf{y}_M(n-1)]_M^{-k+1} \\ \mathbf{y}_{cp}(n) \\ [\mathbf{y}_M(n)]_{M+Q-k-1}^{1} \end{bmatrix},$$

where $k = 0, 1, ..., Q-1$. It can be shown that

$$\bar{\mathbf{y}}_k(n) = \bar{\mathbf{H}}_Q \bar{\mathbf{u}}_k(n) + \text{noise} \tag{1}$$

where

$$\bar{\mathbf{H}}_Q = \begin{bmatrix} \mathbf{H}_{cir} & & \mathbf{0}_{M\times(M+Q-1)} \\ \mathbf{0}_{(L+Q-1)\times(M-L)} & \mathcal{H}_{L+Q-1} & \mathbf{0}_{(L+Q-1)\times(M-L)} \\ \mathbf{0}_{M\times(M+Q-1)} & & \mathbf{H}_{cir2} \end{bmatrix}, \tag{2}$$

and

$$\bar{\mathbf{u}}_k(n) = \begin{bmatrix} [\mathbf{u}_M(n-1)]_M^{-k+1} \\ [\mathbf{u}'_M(n)]_{M+Q-k-1}^{1} \end{bmatrix}. \tag{3}$$

Here, $\mathbf{H}_{cir}$ is an $M \times M$ circulant matrix [8] whose first column is $\begin{bmatrix} \mathbf{h}^T & \mathbf{0}_{M-L-1}^T \end{bmatrix}^T$, $\mathbf{H}_{cir2}$ is obtained by permuting the last L columns of $\mathbf{H}_{cir}$ to the front and is still a circulant matrix, and $\mathcal{H}_k := \mathcal{T}_k(\mathbf{h})^T$ is a $k \times (L+k)$ Toeplitz matrix.

2) Construct a $(2M+Q+L-1) \times Q$ matrix

$$\mathbf{Y}_Q(n) = \begin{bmatrix} \bar{\mathbf{y}}_0(n) & \bar{\mathbf{y}}_1(n) & \cdots & \bar{\mathbf{y}}_{Q-1}(n) \end{bmatrix}.$$

It follows immediately from (1) that

$$\mathbf{Y}_Q(n) = \bar{\mathbf{H}}_Q \mathbf{U}_Q(n) + \text{noise} \tag{4}$$

where

$$\mathbf{U}_Q(n) = \begin{bmatrix} \bar{\mathbf{u}}_0(n) & \bar{\mathbf{u}}_1(n) & \cdots & \bar{\mathbf{u}}_{Q-1}(n) \end{bmatrix} \tag{5}$$

is a $(2M+Q-1) \times Q$ matrix.

3) For consecutive J blocks $\mathbf{y}(0)$, $\mathbf{y}(1)$, ... $\mathbf{y}(J-1)$, construct the $(2M+Q+L-1) \times Q(J-1)$ matrix

$$\mathbf{Y}_Q^{(J)} = \begin{bmatrix} \mathbf{Y}_Q(1) & \mathbf{Y}_Q(2) & \cdots & \mathbf{Y}_Q(J-1) \end{bmatrix}. \tag{6}$$

Then we have

$$\mathbf{Y}_Q^{(J)} = \bar{\mathbf{H}}_Q \mathbf{U}_Q^{(J)} + \text{noise}$$

where

$$\mathbf{U}_Q^{(J)} = \begin{bmatrix} \mathbf{U}_Q(1) & \mathbf{U}_Q(2) & \cdots & \mathbf{U}_Q(J-1) \end{bmatrix} \tag{7}$$

is a $(2M+Q-1) \times Q(J-1)$ matrix.

4) Assume $\mathbf{U}_Q^{(J)}$ has full row rank $2M+Q+L-1$ and perform SVD on $\mathbf{Y}_Q^{(J)}$ so that

$$\mathbf{Y}_Q^{(J)} = \begin{bmatrix} \mathbf{U}_s & \mathbf{U}_n \end{bmatrix} \begin{bmatrix} \mathbf{\Sigma}_s & \mathbf{0} \\ \mathbf{0} & \mathbf{\Sigma}_n \end{bmatrix} \begin{bmatrix} \mathbf{V}_s^\dagger \\ \mathbf{V}_n^\dagger \end{bmatrix}$$

where the diagonal entries of $\mathbf{\Sigma}_n$ are the L smallest singular values of $\mathbf{Y}_Q^{(J)}$.

5) Construct the $(2M+Q-1)L \times (L+1)$ matrix $\mathcal{G}$ using elements of $\mathbf{U}_n$ as described in Section III-A of [10].

6) Let $\hat{\mathbf{h}}$ be the eigenvector of $\mathcal{G}^\dagger \mathcal{G}$ associated with the smallest eigenvalue. This is the estimated channel vector within a scalar ambiguity.

Although the algorithm described above does not require exact knowledge of the values of $\mathbf{s}(n)$ and $\mathbf{R}$, an assumption is made in Step 4 that $\mathbf{U}_Q^{(J)}$ has full row rank. This is the *persistency of excitation* (PE) property of the algorithm stated above. The conditions under which this PE assumption is true, rather than the algorithm itself, are the main focus of this paper.

III. Theoretical Results

A. A Necessary Condition for $\mathbf{U}_Q^{(J)}$ to Have Full Row Rank

A necessary condition for $\mathbf{U}_Q^{(J)}$ to have full row rank is described below.

Lemma 1: $\mathbf{U}_Q^{(J)}$ has full row rank $(2M+Q-1)$ only if

$$(J-2)Q \geq 2M-1. \tag{8}$$

Proof: Observe that $\mathbf{U}_Q^{(J)}$ has $(2M+Q-1)$ rows and $Q(J-1)$ columns. The lemma is readily verified by recognizing that the number of columns cannot be less than the number of rows in order to make a matrix full row rank. ■

Lemma 1 imposes a restriction on the choices of parameters J and Q for the blind algorithm. However, a choice of J and Q satisfying (8) does not automatically guarantee $\mathbf{U}_Q^{(J)}$ having full rank since it depends on the contents of $\mathbf{U}_Q^{(J)}$, which ultimately are dependent on the source data $\mathbf{s}(n)$ and the precoder matrix $\mathbf{R}$. As long as the contents of $\mathbf{s}(n)$ are chosen from a finite constellation $\mathcal{S}$, there is always a nonzero probability that $\mathbf{U}_Q^{(J)}$ is rank-deficient. To see this, simply consider the extreme case where the contents of $\mathbf{U}_Q^{(J)}$ are always chosen as identical values. Since there is no guaranteed conditions for $\mathbf{U}_Q^{(J)}$ to have full rank, it would be of interest to study the probability of $\mathbf{U}_Q^{(J)}$ having full rank given signal constellation $\mathcal{S}$, precoder $\mathbf{R}$, and the values of J and Q.

B. The Probability of $\mathbf{U}_Q^{(J)}$ having full rank

Definition 1: Consider a finite constellation $\mathcal{S} \subset \mathbb{C} - \{0\}$ (which has at least two elements) and an $M \times M$ nonsingular precoder $\mathbf{R} \in \mathbb{C}^{M \times M}$. Let each element of the $M \times J$ matrix $\mathbf{S} = \begin{bmatrix} \mathbf{s}(0) & \mathbf{s}(1) & \cdots & \mathbf{s}(J-1) \end{bmatrix}$ be independently selected from the constellation $\mathcal{S}$ with equal probabilities for each element in $\mathcal{S}$. Let $\mathbf{u}_M(n) = \mathbf{R}\mathbf{s}(n)$ and let $\mathbf{U}_Q^{(J)}$ be defined as in Eq. (7). For $J \geq 2, Q \geq 1$, the probability that $\mathbf{U}_Q^{(J)}$ has full rank will be denoted as $P_{\mathcal{S},\mathbf{R}}(J, Q)$. ∎

Obviously, $P_{\mathcal{S},\mathbf{R}}(J, Q) = 0$ whenever $(J-2)Q < 2M-1$ since (8) is violated. From the discussions above, we have $P_{\mathcal{S},\mathbf{R}}(J, Q) < 1$ for all J and Q as long as $\mathcal{S}$ is a finite set. The probability, however, can be increased by increasing J. It can be shown that

$$P_{\mathcal{S},\mathbf{R}}(J+1, Q) \geq P_{\mathcal{S},\mathbf{R}}(J, Q) \tag{9}$$

since the row rank of a matrix never decreases when additional columns are appended. Furthermore, it can be proved that

$$\lim_{J \to \infty} P_{\mathcal{S},\mathbf{R}}(J, Q) = 1$$

for any constellation $\mathcal{S}$ and precoder $\mathbf{R}$ (and any $Q \geq 1$). However, increasing J means more data needs to be accumulated at the receiver which causes a delay in channel estimation. In addition, in the case when a time-varying channel is present, a larger J makes the channel estimate less meaningful since the channel coefficients may have changed significantly while J blocks are being accumulated.

Increasing Q, on the other hand, does not require additional data and hence does not have the drawback described above. It can be shown that increasing Q also helps increase the value of $P_{\mathcal{S},\mathbf{R}}(J, Q)$. Using the following theorem, it can be shown that

$$P_{\mathcal{S},\mathbf{R}}(J, Q+1) \geq P_{\mathcal{S},\mathbf{R}}(J, Q). \tag{10}$$

Theorem 1: If $\mathbf{U}_Q^{(J)}$ has full row rank $(2M+Q-1)$, then $\mathbf{U}_{Q+1}^{(J)}$ also has full row rank $(2M+Q)$.

Proof: Assume $\mathbf{U}_{Q+1}^{(J)}$ does not have full row rank while $\mathbf{U}_Q^{(J)}$ does. Then there exists a nonzero row vector $\mathbf{v}^T = \begin{bmatrix} v_1 & \cdots & v_{2M+Q} \end{bmatrix}$ such that $\mathbf{v}^T\mathbf{U}_{Q+1}^{(J)} = \mathbf{0}^T$. From the definition in Eq. (7), we obtain that $\mathbf{v}^T$ is a left annihilator of $\mathbf{U}_{Q+1}(n)$ for $1 \leq n \leq J-1$. The notation of $\mathbf{U}_Q(n)$ was defined in Eq. (5). Notice that $\mathbf{U}_Q(n)$ is a submatrix of $\mathbf{U}_{Q+1}(n)$ and can be obtained by removing the first row and the first column of $\mathbf{U}_{Q+1}(n)$, or by removing the last row and the last column of $\mathbf{U}_{Q+1}(n)$. This means that both $\mathbf{v}_1^T = \begin{bmatrix} v_1 & \cdots & v_{2M+Q-1} \end{bmatrix}$ and $\mathbf{v}_2^T = \begin{bmatrix} v_2 & \cdots & v_{2M+Q} \end{bmatrix}$ are left annihilators of $\mathbf{U}_Q(n)$ for $1 \leq n \leq J$. So $\mathbf{v}_1^T\mathbf{U}_Q^{(J)} = \mathbf{v}_2^T\mathbf{U}_Q^{(J)} = \mathbf{0}^T$. Since $\mathbf{v}^T$ is nonzero, at least one of $\mathbf{v}_1^T$ and $\mathbf{v}_2^T$ must also be nonzero. This implies that $\mathbf{U}_Q^{(J)}$ does not have full rank and contradicts the assumption. ∎

Although increasing Q never decreases $P_{\mathcal{S},\mathbf{R}}(J, Q)$, $P_{\mathcal{S},\mathbf{R}}(J, Q)$ can never approach unity even when $Q \to \infty$. The probability of $\mathbf{U}_Q^{(J)}$ having full rank always stops increasing when $Q \geq 2M-1$. This is a consequence of the following theorem.

Theorem 2: If $\mathbf{U}_Q^{(J)}$ does not have full rank when $Q = 2M-1$, then $\mathbf{U}_Q^{(J)}$ does not have full rank for any Q.

Proof: See [11]. ∎

Combining Theorems 1 and 2, we immediately have

$$P_{\mathcal{S},\mathbf{R}}(J, Q) = P_{\mathcal{S},\mathbf{R}}(J, 2M-1)$$

for any $Q \geq 2M-1$. Now, given $\mathcal{S}$, $\mathbf{R}$, and $J \geq 3$, the value $P_{\mathcal{S},\mathbf{R}}(J, Q)$ is zero when $Q < (2M-1)/(J-2)$. $P_{\mathcal{S},\mathbf{R}}(J, Q)$ first becomes nonzero when $Q = \lceil (2M-1)/(J-2) \rceil$. From now on, $P_{\mathcal{S},\mathbf{R}}(J, Q)$ may keep increasing until $Q = 2M-1$, when $P_{\mathcal{S},\mathbf{R}}(J, Q)$ reaches its upper bound. In summary,

$$\begin{aligned} 0 &= P_{\mathcal{S},\mathbf{R}}(J, Q)\,|_{1 \leq Q < Q_{min}} < P_{\mathcal{S},\mathbf{R}}(J, Q_{min}) \\ &\leq P_{\mathcal{S},\mathbf{R}}(J, 2M-1) = P_{\mathcal{S},\mathbf{R}}(J, Q)|_{Q \geq 2M-1} < 1, \end{aligned}$$

where $Q_{min} := \lceil (2M-1)/(J-2) \rceil$.

IV. Numerical Simulations

In the previous section we understand that the value of $P_{\mathcal{S},\mathbf{R}}(J, Q)$ increases in general as J or Q increases. However, we have not had a chance to look at the real values of $P_{\mathcal{S},\mathbf{R}}(J, Q)$. Since a close-form expression of $P_{\mathcal{S},\mathbf{R}}(J, Q)$ is difficult to obtain, we perform numerical simulations to obtain approximate values of $P_{\mathcal{S},\mathbf{R}}(J, Q)$. The simulations are performed with three commonly used constellations in communications: BPSK ($\mathcal{S} = \{\pm 1\}$), QPSK ($\mathcal{S} = \{\pm 1, \pm j\}$), and 16-QAM ($\mathcal{S} = \{a + bj | a, b \in \{\pm 1, \pm 3\}\}$). The $M \times M$ precoder $\mathbf{R}$ is chosen as $\mathbf{I}_M$ for SC-CP systems and $\mathbf{W}^\dagger$ for OFDM systems. Although the exact probability of $\mathbf{U}_Q^{(J)}$ having full rank can be actually obtained by testing all possible transmitted data, an exhaustive simulation is barely feasible. For each $J \geq 3$, the simulations are performed for two values of Q: $Q = 2M-1$ and $Q = \lceil (2M-1)/(J-2) \rceil$. More than 1,000 independent realizations of $\mathbf{U}_Q^{(J)}$ were used to evaluate the value $P_{\mathcal{S},\mathbf{R}}(J, Q)$ for any given $\mathcal{S}$, $\mathbf{R}$, J, and Q. When $Q = 2M-1$, the simulation gives an upper bound of $P_{\mathcal{S},\mathbf{R}}(J, Q)$ for a given J and the simulation where $Q = \lceil (2M-1)/(J-2) \rceil$ gives a lower bound of nonzero $P_{\mathcal{S},\mathbf{R}}(J, Q)$. M is chosen as 16.

Figures 2 and 3 show the results when the precoder $\mathbf{R}$ is chosen as an identity matrix $\mathbf{I}_M$ (SC-CP) and a normalized

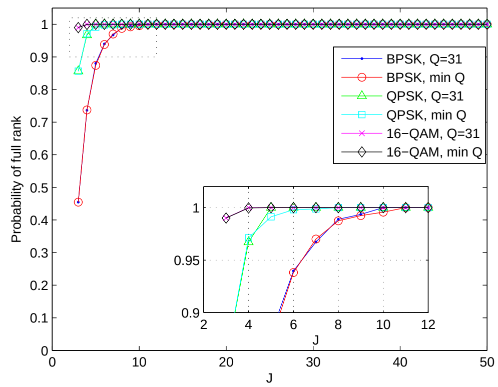

Fig. 2. The probability of $\mathbf{U}_Q^{(J)}$ having full rank in SC-CP systems.

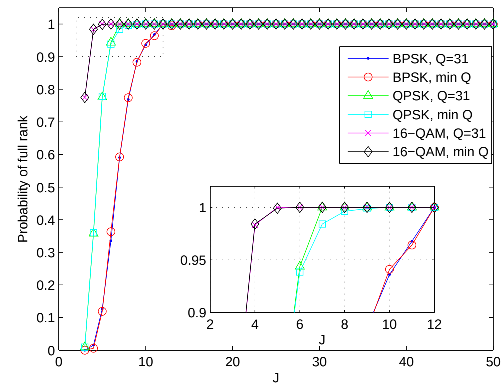

Fig. 3. The probability of $\mathbf{U}_Q^{(J)}$ having full rank in OFDM systems.

IDFT matrix $\mathbf{W}^{\dagger}$ (OFDM), respectively. Some comments on these results are made below.

1) As expected, the probability of $\mathbf{U}_Q^{(J)}$ having full rank is smaller when a smaller constellation is used or when J is smaller. When $J \geq 12$, the probability becomes very close to unity for all combinations of constellations and precoders. When a 16-QAM constellation is used, the probability is already very high when $J = 5$.
2) It should be especially noted that the probability of $\mathbf{U}_Q^{(J)}$ having full rank is significantly smaller when $\mathbf{R}$ is chosen as the IDFT matrix than when $\mathbf{R}$ is an identity matrix. An explanation of this phenomenon can be found in [11]. This phenomenon suggests the algorithm proposed in [10] is more stable when operated in SC-CP systems than in OFDM systems when the constellation is small and/or when J is small.
3) Finally, although the theory suggests
$$P_{\mathcal{S},\mathbf{R}}\left(J, 2M-1\right) \geq P_{\mathcal{S},\mathbf{R}}\left(J, \left\lceil \frac{2M-1}{J-2} \right\rceil\right),$$
in simulation the above two quantities look almost the same so that a conjecture may be made that
$$P_{\mathcal{S},\mathbf{R}}\left(J, Q\right) = P_{\mathcal{S},\mathbf{R}}\left(J, \left\lceil \frac{2M-1}{J-2} \right\rceil\right)$$
for any $Q \geq \lceil (2M-1)/(J-2) \rceil$. This conjecture, however, has not yet been verified or disproved.

V. Conclusions

In this paper we studied the persistency of excitation (PE) property in a recently reported blind channel estimation algorithm in cyclic prefix (CP) systems, which is an essential property for the algorithm to work properly. Specifically, the probability of the special-structured matrix $\mathbf{U}_Q^{(J)}$ to have full rank is studied for different constellations, precoders, numbers of blocks J, and repetition indices Q. Theoretical derivations as well as numerical simulations indicate that the probability of PE converges to unity when $J \rightarrow \infty$, but it stops increasing with Q if $Q \geq 2M-1$. In addition when the precoder $\mathbf{R}$ is chosen as an IDFT matrix, the probability of PE is much smaller than when $\mathbf{R}$ is chosen as the identity matrix. This suggests higher stability for the recently reported algorithm to work on single-carrier-cyclic-prefix (SC-CP) systems than on OFDM systems, especially when the number of blocks J is small.

In the future it remains of interest to theoretically prove or disprove the conjecture raised in remark 3) of Section IV.

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