ZERO-FORCING COSINE MODULATED FILTERBANK TRANSCEIVERS WITH CYCLIC PREFIX

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ABSTRACT

The DMT transceivers have enjoyed great success in high speed data transmission. The DMT scheme is realized by a transceiver that divides the frequency selective channel into a set of frequency nonselective parallel subchannels. The efficiency of the scheme depends on the frequency selectivity of the transmitting and receiving filters. In the past, the filterbank transceiver has been proposed as an implementation of DMT transceiver that has better frequency band separation. But usually ISI can not be completely cancelled in these filterbank transceivers and additional equalization is required. In this paper, we show how to obtain zero-forcing cosine modulated filterbank transceivers by introducing cyclic prefix at the transmitter. The proposed systems have the advantages of having good frequency separation and simple equalization. ISI can easily be elimination by exploiting the cyclic prefix and good frequency band separation is attained through the use of cosine modulated filter banks.

1. INTRODUCTION

Discrete Multitone modulation (DMT) has been successfully employed for high speed data transmission over frequency selective channels such as DSL [1]. Fig. 1(a) shows an N-band DFT-based DMT system. By inserting a cyclic prefix that is long enough, the frequency selective channel is converted into N frequency non-selective parallel subchannels. The input bit stream is parsed and coded as modulation symbols, e.g., PAM or QAM. By judiciously allocating the bits among the subchannels, high speed data transmission can be achieved using DFT-based DMT system at a relatively low cost [1]. In a DFT-based DMT system, IDFT and DFT matrices are used as the modulation and demodulation matrices. It is known that DFT filters have a large spectral overlap and their stopband attenuation is 13 dB only. In the presence of narrowband noise or highly colored noise, the performance of DFT-based DMT system will degrade significantly due to its poor frequency band separation.

For better band separation, Sandberg and Tzannes [2] proposed the so called discrete wavelet multitone (DWMT) system, in which perfect reconstruction filter bank (FB) is used as the transceiver. The transmitting and receiving filters have excellent frequency separation property inherited from good filter bank designs. For frequency selective channel, there is inter-band as well as intra-band interferences in DWMT systems. Unlike the DFT-based DMT system, there is no simple equalization technique for DWMT systems. Performance evaluation conducted in [3][4] shows that the resulting ISI can seriously degrade the system performance. To reduce the amount of ISI, inter- and intra-subband equalization are performed on the receiver outputs in [2][5].

In this paper, we introduce a new FB transceiver structure using cosine modulated filter bank (CMFB). Like DFT-based DMT systems, the proposed structure combats ISI by adding cyclic prefix (CP) at the transmitter. Due to the added CP, the frequency selective channel is converted into a parallel of frequency non selective channels. And simple frequency equalization technique can be used to cancel ISI. The new CMFB transceiver with cyclic prefix (CP-CMFB) can be viewed as a combination of the CMFB transceiver and the single carrier modulation with cyclic prefix (SC-CP) [6]. The SC-CP system enjoys the advantage of simple equalization but it suffers from having no frequency separation ability. The CMFB transceiver on the other hand has excellent frequency separation but there is no simple equalization technique. The proposed CP-CMFB transceiver can exploit the advantages of both SC-CP and CMFB transceivers. In the CP-CMFB transceiver, ISI is cancelled using SC-CP and frequency separation is attained through the use of CMFB. Simulation shows that the CP-CMFB transceiver outperforms both the DFT-based DMT and SC-CP transceivers.

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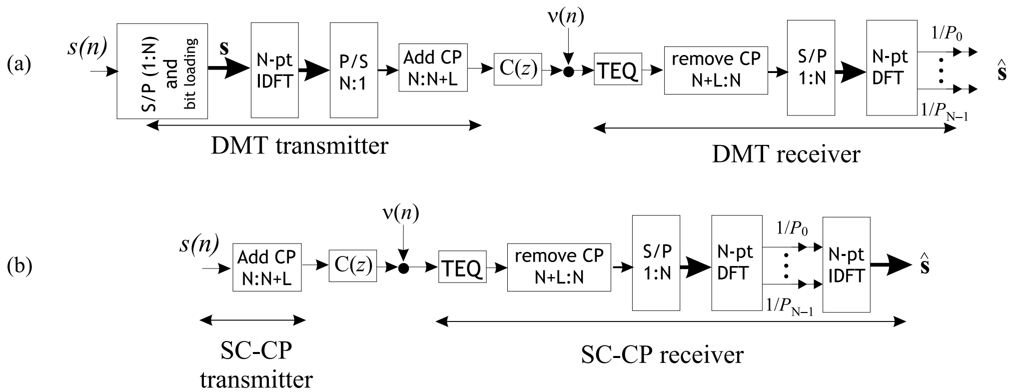

Figure 1: (a) DFT-based DMT scheme. (b) Single carrier with cyclic prefix scheme.

2. REVIEW OF DMT AND SINGLE CARRIER SYSTEMS WITH CYCLIC PREFIX

Fig. 1(a) shows the block diagram for the DFT-based DMT system. At the transmitter, the input data stream is parsed into a $N \times 1$ vector $\mathbf{s}$ consisting of QAM or PAM symbols. Then N-point IDFT is performed. For each block of N samples, a cyclic prefix (CP) of length L is added. Therefore the redundancy is given by L/N. In practice, N is usually considerably larger than L. Hence the redundancy is in general small. In this paper, $N = 512$ and $L = 32$. The channel is assumed to be a LTI filter $C(z)$. To reduce the redundancy, a time-domain equalizer (TEQ) is in general applied at the receiver to shorten the length of the effective channel impulse response. Let the TEQ-equalized effective channel be $P(z)$ whose length is less than L. The channel noise $\nu(n)$ is assumed to be a WSS process. At the receiver, the L samples of CP are first removed from every block of $(N + L)$ received data. Then N-point DFT is then performed. The set of scalars $1/P_k$ are known as the frequency domain equalizers (FEQ), where $P_0, \cdots, P_{N-1}$ are the N-point DFT of the channel impulse response $p(n)$. It is known that such a system is zero-forcing; that is, in the absence of channel noise, $\hat{\mathbf{s}} = \mathbf{s}$. Let the output noise vector $\mathbf{e} = \hat{\mathbf{s}} - \mathbf{s}$. Then bits can be judiciously allocated among the subchannel so that the performance is optimized. More precisely, the number of bits assigned to the kth subchannel should be

$$b_k = b + 0.5 \log_2 \sigma_k^2 - 0.5 \log_2 \sigma_{GM}^2, \tag{1}$$

where σ_k^2 is the variance of e_k and σ_{GM}^2 is their geometrical mean. The quantity b is the bit rate and it is given as $b = 1/N \sum_{k=0}^{N-1} b_k$. Given the sampling period T_s in the ADC and DAC, then the transmission rate R (bps) is related to the bit rate as

$$R = \left(\frac{N}{N+L}\right)\frac{b}{T_s}.$$

The factor $N/(N+L)$ is due to the fact that for every N data samples, a cyclic prefix of length L is added. It is known that the performance gain provided by the optimal bit allocation is equal to the ratio $\sigma_{AM}^2/\sigma_{GM}^2$, where σ_{AM}^2 is the arithmetic mean of σ_k^2. When the spectrum of channel noise $\nu(n)$ is highly nonflat, this gain can be substantial if the receiving filters have a good frequency band separation. For DFT-based DMT system, this gain is limited due to its poor frequency band separation.

In [6], the idea of cyclic prefix is introduced in the single carrier transmission systems. The block diagram of the single carrier system with cyclic prefix (SC-CP) is shown in Fig. 1(b). At the transmitter, a CP of length L is added to every block of N data samples. By adding CP, the receiver can carry out a simple frequency equalization technique as shown in Fig. 1(b) and zero forcing transceiver can be achieved. The SC-CP system can be obtained from the DFT-based DMT system by moving the IDFT operation to the receiver and therefore its complexity is the same as the DFT-based DMT system. Note that the transmitting filters of the SC-CP system do not have any frequency separation ability. Though it was demonstrated [7] that for wireless transmission or broadcasting applications where bit loading is not applicable, the SC-CP system outperforms the DFT-based DMT system (also known as the OFDM). In fact, it was proved that in the absence of bit loading, the SC-CP system is the optimal transceiver that minimizes the bit error rate. However for applications where the transmission environment varies slowly such as DSLs, efficient bit loading can be done. For these applications, the SC-CP transceiver is not a suitable choice. The reason is as follows. For SC-CP systems, it was shown [7] that the noise variances σ_k^2 are the same for all k. Therefore bit allocation cannot provide any gain in SC-CP systems.

3. ZERO-FORCING CMFB TRANSCEIVERS WITH CYCLIC PREFIX

Fig. 2 shows an M-channel FB system. When the output of a FB is always equal to its input, we say that the

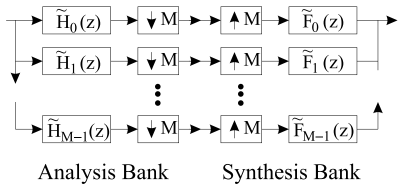

Figure 2: Analysis and synthesis banks of a FB system.

FB has perfect reconstruction (PR). FB systems have found many applications [8] such as signal compression, denoising, etc. In particular, the class of cosine modulated filter bank (CMFB) have drawn a lot of attention due to its low complexity. In a CMFB, the synthesis filters $\tilde{F}_k(z)$ are the cosine-modulated versions of a prototype filters (so are the analysis filters $\tilde{H}_k(z)$). Thus the cost of the synthesis or analysis bank reduces to that of a prototype filter and an $M \times M$ DCT matrix. There are many effective design methods for CMFBs with a good frequency band separation ([8][9] and references therein). In this paper, we will use a 32-channel CMFB with prototype filter length $l_0 = 256$ designed by Nguyen [9]. The coefficients of the prototype filter are obtained from http://saigon.ece.wisc.edu. The magnitude responses of the first 3 filters are shown in Fig. 3. The stopband attenuation is more than 40 dB. It has a much better frequency band separation than the DFT filters. Note that the number of bands is $M = 32$, a relative small number compared with the DMT case where $N = 512$. This is because of two reasons. Firstly, CMFBs with a large number of bands are difficult to design, as demonstrated by the numerical examples given in http://saigon.ece.wisc.edu. Secondly, the implementation cost of CMFB with a large number of bands is very high.

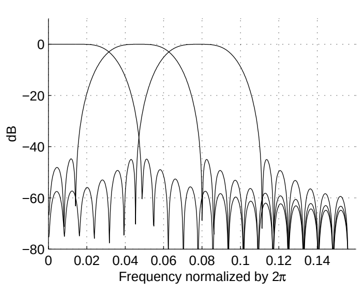

Figure 3: Magnitude response of the first 3 filters of a 32-channel CMFB.

When the synthesis and analysis banks are used as the transmitter and the receiver respectively, we have a transceiver system. For frequency selective channels, the FB transceiver will suffer from severe ISI. Either a time-domain linear equalizer or a subband equalizer is needed to equalize the channel. In many applications such as DSLs, the filter order of linear equalizer needed to achieve a satisfactory performance is very long and it becomes very costly, as we will see later in the numerical example. Subband equalizer on the other hand needs to cancel inter and intra band interference and its complexity is also high[2].

In this paper, a different approach is employed. Like the SC-CP system, we introduce cyclic prefix at the transmitter to combat the ISI. The resulting CMFB with cyclic prefix scheme (CP-CMFB) is shown in Fig. 4. When the length of the CP, L is longer than the TEQ equalized channel, the CP-CMFB transceiver in Fig. 4 will achieve zero forcing. To reduce the redundancy, instead of adding 32 CP samples to every block of 32 data samples, we add 32 CP samples to every block of 512 data samples. Hence the redundancy is the same as that of the DMT scheme showed in Fig. 1.

Equivalent Transmitting and Receiving Filters

Note that due to the CP insertion and CP removal, the transmitting filters and receiving filters are no longer the synthesis and analysis filters respectively. For notational simplicity, we choose $N = \nu M$ and $M = L$. Let the $M \times M$ analysis and synthesis polyphase matrices of the CMFB be $\mathbf{A}(z)$ and $\mathbf{S}(z)$ respectively. Decomposing these matrices into their polyphase component with respect to ν, we get:

$$\mathbf{S}(z) = \sum_{i=0}^{\nu-1} \mathbf{S}_i(z^\nu) z^{-i}, \quad \mathbf{A}(z) = \sum_{i=0}^{\nu-1} \mathbf{A}_i(z^\nu) z^{-i}. \quad (2)$$

Applying multirate identities [8] and after some algebraic simplifications (not difficult but tedious), one can show that the CP-CMFB transceivers in Fig. 4 can be redrawn as the FB transceiver shown in Fig. 5. The $(N+L)\times N$ polyphase matrix for the transmitting filter $F_k(z)$ is given by

$$\mathbf{R}(z) = \begin{pmatrix} \mathbf{S}_{\nu-1}(z) & \mathbf{S}_{\nu-2}(z) & \dots & \mathbf{S}_0(z) \\ \mathbf{S}_0(z) & z^{-1}\mathbf{S}_{\nu-1}(z) & \dots & z^{-1}\mathbf{S}_1(z) \\ \mathbf{S}_1(z) & \mathbf{S}_0(z) & \dots & z^{-1}\mathbf{S}_2(z) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{S}_{\nu-1}(z) & \mathbf{S}_{\nu-2}(z) & \dots & \mathbf{S}_0(z) \end{pmatrix}. \quad (3)$$

Note that the top row $[\mathbf{S}_{\nu-1}(z) \;\; \mathbf{S}_{\nu-2}(z) \;\; \dots \;\; \mathbf{S}_0(z)]$ is identical to the last row and this is due to the CP insertion. The transmitting filters in Fig. 5 are given by

$$[F_0 \; \dots \; F_{N-1}] = [1 \; z^{-1} \; \dots \; z^{-N-L+1}]\mathbf{R}(z^{(N+L)}). \quad (4)$$

To see how these transmitting filters related to the original CMFB synthesis filters $\tilde{F}_i(z)$, we take the example of $N = 512, \; M = 32, \; L = 32$, and the length of the original CMFB $l_0 = 256$ (which is also equal to the length of the CMFB prototype). In this case, $\nu = N/M = 16$ and the 32×32 polyphase matrix of the original CMFB synthesis filters, $\mathbf{S}(z)$ in (2), has order equal to $(l_0/M - 1) = 7$. In other words, $\mathbf{S}_0 \sim \mathbf{S}_7$ in (2) are constant matrices independent of z and $\mathbf{S}_8 \sim \mathbf{S}_{15}$ are zero matrices. From (3), we see that the CP operation adds only zeros to the first 256

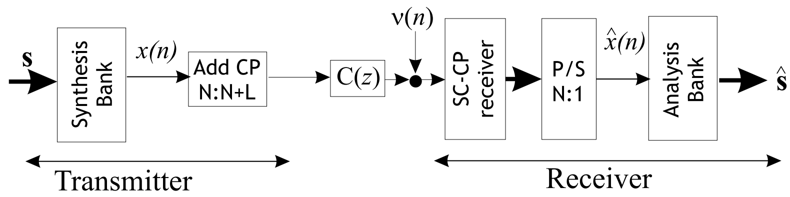

Figure 4: The proposed CMFB transceiver with cyclic prefix.

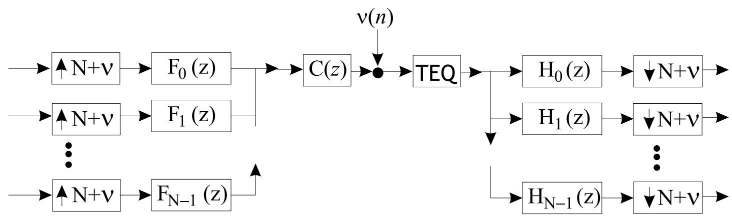

Figure 5: An equivalent structure for the proposed CP-CMFB transceiver

columns in $\mathbf{R}(z)$. Using (4), we conclude that $F_0 \sim F_{255}$ have identical magnitude responses as the original CMFB synthesis filters. For $F_{256} \sim F_{511}$, parts of their impulse responses are folded to the front, their magnitude responses will be affected by the CP operation. Fig. 6 shows the responses for two sets of transmitting filters. One can see that $F_{128} \sim F_{159}$ have the same responses as the original CMFB synthesis filters while $F_{384} \sim F_{415}$ have essentially no stopband attenuation. However this will not affect the transceiver performance as the bit error rate performance is determined by the receiver noise variances, which is affected only by the receiving filters.

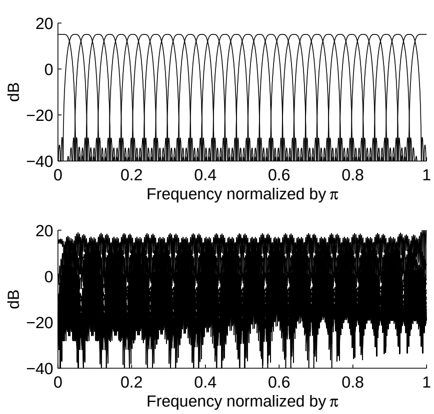

Figure 6: Magnitude responses of $F_{128} \sim F_{159}$ (top) and $F_{384} \sim F_{415}$ (bottom)

To derive the receiving filters $H_k(z)$, we define the $N \times N$ pseudo-circulant matrix:

$$\mathbf{A}_{big}(z) = \begin{pmatrix} \mathbf{A}_0(z) & z^{-1}\mathbf{A}_{\nu-1}(z) & \dots & z^{-1}\mathbf{A}_1(z) \\ \mathbf{A}_1(z) & \mathbf{A}_0(z) & \dots & z^{-1}\mathbf{A}_2(z) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{A}_{\nu-1}(z) & \mathbf{A}_{\nu-2}(z) & \dots & \mathbf{A}_0(z) \end{pmatrix}.$$

Then the polyphase matrix for the receiving filters is

$$\mathbf{E}(z) = \mathbf{A}_{big}(z)\mathbf{W}^H\mathbf{\Lambda}^{-1}\mathbf{W}[\mathbf{0}_{N\times L}\ \mathbf{I}],$$

where $\mathbf{W}$ is the $N \times N$ DFT matrix, $\mathbf{\Lambda}^{-1}$ is the matrix corresponding to the FEQ and the matrix $[\mathbf{0}_{N\times L}\ \mathbf{I}]$ describes the CP removal. The receiving filters in Fig. 5 are given by

$$[H_0\ \dots\ H_{N-1}]^T = \mathbf{E}(z^{(N+L)})[1\ z\ \dots\ z^{N+L-1}]^T.$$

Note that the responses of the receiving filters are affected by both the channel through the matrix $\mathbf{W}^H\mathbf{\Lambda}^{-1}\mathbf{W}$ and the CP removal. The action of the matrix $\mathbf{W}^H\mathbf{\Lambda}^{-1}\mathbf{W}$ can be interpreted as a reshaping of the passband of H_i so that the magnitude gain of their passband approximates the inverse of the TEQ equalized channel $P(z)$. For the CP operation, following a similar derivation as the case of transmitting filter, one can verify that $H_0 \sim H_{255}$ are affected and $H_{256} \sim H_{511}$ are not affected by the CP operation. In Fig. 7, we have plotted magnitude responses of the receiving filters $H_{128} \sim H_{159}$ and $H_{384} \sim H_{415}$. The parameters are defined in the simulation in Sec. 4. From the plots, we see that the responses $H_{384} \sim H_{415}$ have a stopband attenuation of more than 40 dB and they resemble that of the original CMFB in Fig. 3. Note that the passbands of these filters are no longer flat as the receiving filters need to equalize the channel. Comparing the shape of these passbands and the magnitude response of the TEQ equalized channel $P(z)$ given in Fig. 8(a), we see that they are nearly reciprocal of each other. For $H_{128} \sim H_{159}$, their stopband attenuation is around 20 dB only because they are affected by the CP removal.

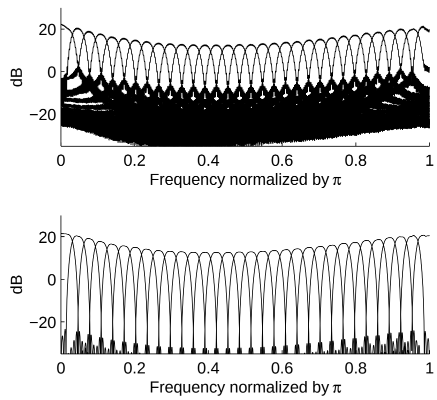

Figure 7: Magnitude responses of $H_{128} \sim H_{159}$ (top) and $H_{384} \sim H_{415}$ (bottom)

From the above analysis, we see that though the equivalent transceiver has $N = 512$ bands, the CMFB divides the frequency into $M = 32$ bands only. Many transmitting and receiving filters will occupy the same frequency region. Though some of the receiving filters are affected by the CP operation, their responses are still better than that of the DFT filters. Thus they will have a better performance than the DFT-based DMT system as we will demonstrate next.

4. SIMULATION RESULTS

The TEQ-equalized channel $P(z)$ (the cascade of the channel $C(z)$ and the TEQ) is shown in Fig. 8(a). The power spectrum for the noise at the TEQ output is given in Fig. 8(b). These parameters are obtained from a typical ADSL environment. The sampling rate T_s is 2 MHz and the transmission rate is fixed at $R = 3$ Mbps. The parameters $M = 32$, $N = 512$ and $L = 32$. Optimal bit allocation in (1) is applied to all the transceiver systems. The simulation assumes that all the receivers have a perfect estimation of the channel response. For DFT-based DMT system, the modulation symbols are QAM with complex conjugate assignment. For the SC-CP and all CMFB systems, the modulation symbols are PAM.

We compare the performance of 5 different transceivers. These systems include the DFT-based DMT system, the SC-CP system and 3 different CMFB transceivers. The first CMFB transceiver is the proposed CP-CMFB system. The second CMFB system employs a linear equalizer minimizing the ISI. The third CMFB transceiver uses a linear equalizer minimizing mean squared error (MMSE) . In the second and third CMFB system, no CP is added so there is no redundancy. We adjust the bit rate so that all 5 transceivers have the same transmission rate of $R = 3$ Mbps.

The bit error rate (BER) performances of these systems are shown in Fig. 9. From the plot, one can see that the SC-CP system has the worst performance. This is due to the fact that its filters have essentially no frequency separation ability. In the experiment it is found that all the bands get the same bit allocation. The DFT-based DMT system outperforms the SC-CP system by more than 2 dB. The proposed CP-CMFB transceiver outperforms the DFT-based DMT system. At $BER = 10^{-5}$, the gain is more than 2 dB. Both the CMFB systems with linear equalizer have a slightly better performance than the CP-CMFB. However the design and implementation cost of the linear equalizers are much higher. In this example, the TEQ has only 5 taps, but to obtain a satisfactory performance, the CMFB systems without CP need linear equalizers with 65 taps (minimizing ISI) and 129 taps (MMSE) respectively.

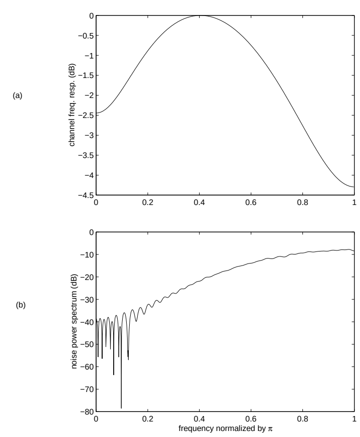

Figure 8: (a) Magnitude response of the TEQ equalized channel $P(z)$; (b) power spectrum of noise at the TEQ output.

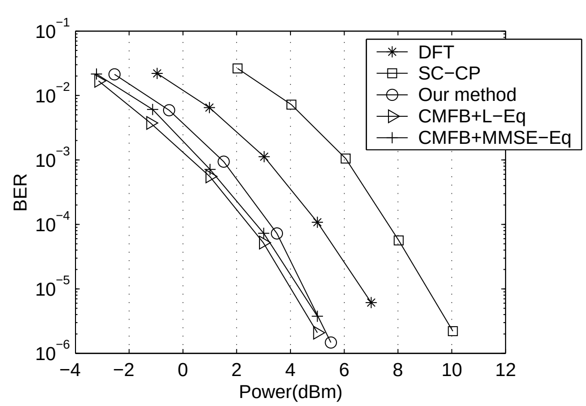

Figure 9: BER comparison.

5. CONCLUSIONS

In this paper, we have introduced a CMFB transceiver with CP. The CP-CMFB transceivers have the advantages of having good frequency separation and simple equalization. ISI can easily be elimination by exploiting the CP and good frequency band separation is attained through the use of CMFB. Because of the good frequency band separation property of the CMFB, we have a gain of 2 dB over the DFT based DMT system.

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