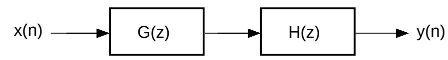
EE112/PPV/Winter'09: Digital Signal Processing

HW SET #9 Last HW. Due On **Tue Mar. 17 at 4:00 PM** outside PPV's office (in Andrea's) office.

Note. There will be no finals.

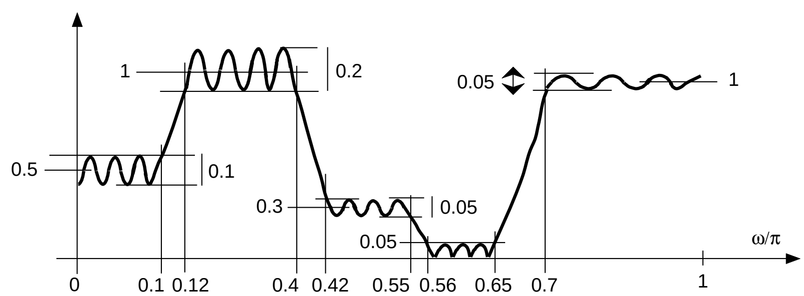
1. (180 points) Consider the following block diagram.



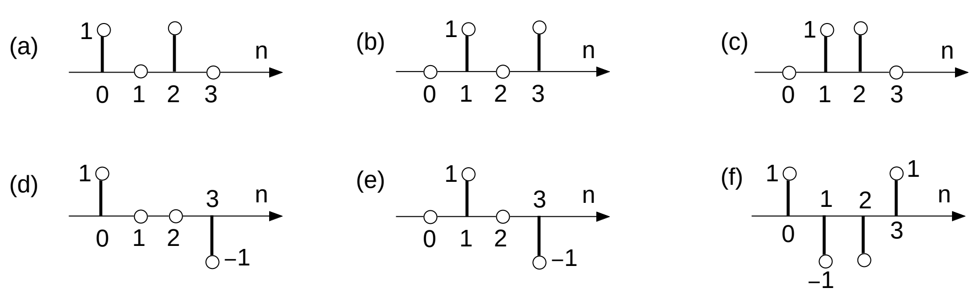
Here $G(z)$ is an unavoidable linear distortion (caused, for example, by a transmission medium) whose response is estimated to be such that

$$|G(e^{j\omega})| = [2 + \cos\omega]/3$$

for $-\pi \le \omega \le \pi$. We wish to design the linear phase filter $H(z)$ such that $Y(e^{j\omega})/X(e^{j\omega})$ has magnitude as shown in the next figure. Note that each band is required to be equiripple and the desired ripple size is indicated. The regions between the rippled bands are transition bands. (The number of ripples shown in each band is arbitrary; do not take them seriously). Write down the "desired response" $D(\omega)$ and the "weighting function" $W(\omega)$ to be used in the Remez exchange algorithm for the design of $H(z)$. Do not worry about finding the order N.



2. (120 points) For this problem you should use the properties of the DFT efficiently. There is no need to evaluate the DFT explicitly. The figure shows several sequences defined for $0 \le n \le 3$. Identify all sequences which have real-valued DFT. Also identify all sequences which have imaginary valued DFT.



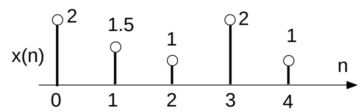
Over ⇒

3. (150 points) Let $X[k]$ denote the N-point DFT of $x(n)$, and define a new sequence

$$Y[\ell] = \sum_{k=0}^{N-1} X[k]W^{\ell k}$$

That is, $Y[\ell]$ is the DFT of DFT of $x(n)$.

a) Express $Y[\ell]$ directly in terms of $x(n)$. For the example of $x(n)$ shown in the figure with $N = 5$, draw $Y[\ell]$ and indicate sample values.



b) Now consider a N-point sequence $V[m]$ defined as

$$V[m] = \text{ DFT of DFT of DFT of DFT of } x(n)$$

Express $V[m]$ directly in terms of $x(n)$. For the example of $x(n)$ shown in the figure, draw $V[m]$ and indicate sample values.

4.** (200 points) Let $H(z) = \sum_{n=0}^{N} h(n)z^{-n}$ be some FIR filter. Let the zeros of this filter be $z_k, 1 \leq k \leq N$. Some of these zeros could be inside, some outside, and some on the unit circle. Suppose we replace every zero outside the unit circle with its reciprocal conjugate, that is replace the factor $(1 - z^{-1}\alpha)$ in $H(z)$ with $(-\alpha^* + z^{-1})$. If we do this for every zero outside the unit circle, then the resulting filter $G(z) = \sum_{n=0}^{N} g(n)z^{-n}$ has all zeros inside and on the unit circle, and none outside. We say that $G(z)$ is a ***minimum-phase system. Any FIR system with no zeros outside the unit circle is said to have minimum phase.

a) Show that $G(z)$ and $H(z)$ have the same magnitude response, that is, $|G(e^{j\omega})| = |H(e^{j\omega})|$. So, only the phases are different.

b) Show that the impulse response $g(n)$ satisfies the following property:

$$\sum_{n=0}^{K} |g(n)|^2 \geq \sum_{n=0}^{K} |h(n)|^2$$

for all integer $K \geq 0$. In this sense, the energy of a minimum phase filter $G(z)$ is more concentrated near zero time, compared to the arbitrary phase filter $H(z)$.

Reading assignments.

1. Review lecture notes.

Reminders:

Late homework policy for EE112. Late homeworks will not be accepted. No exceptions other than institute-established emergency reasons, in which case a signed letter is required from authorized official.

NCT Problems. Remember that problems with an asterik, such as *6 are no-collaboration (NCT) problems.

Books. AVO's book means "Discrete time signal processing" by Oppenheim et al. PPV's book means "Multirate systems and filter banks" by PPV. Most homework problems come from these books.