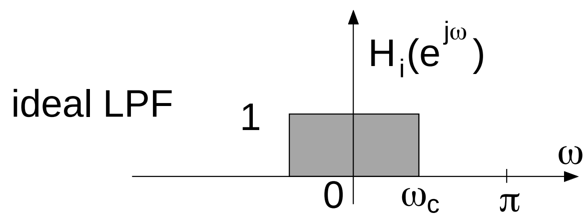
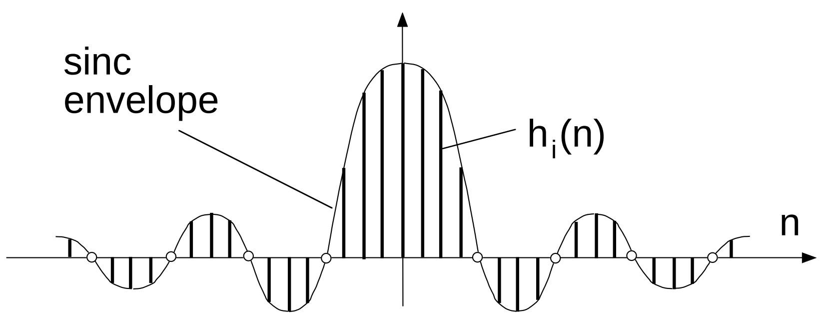
# EE112/PPV/Winter'09
# Digital signal processing

## Handout 5: Window based FIR design:

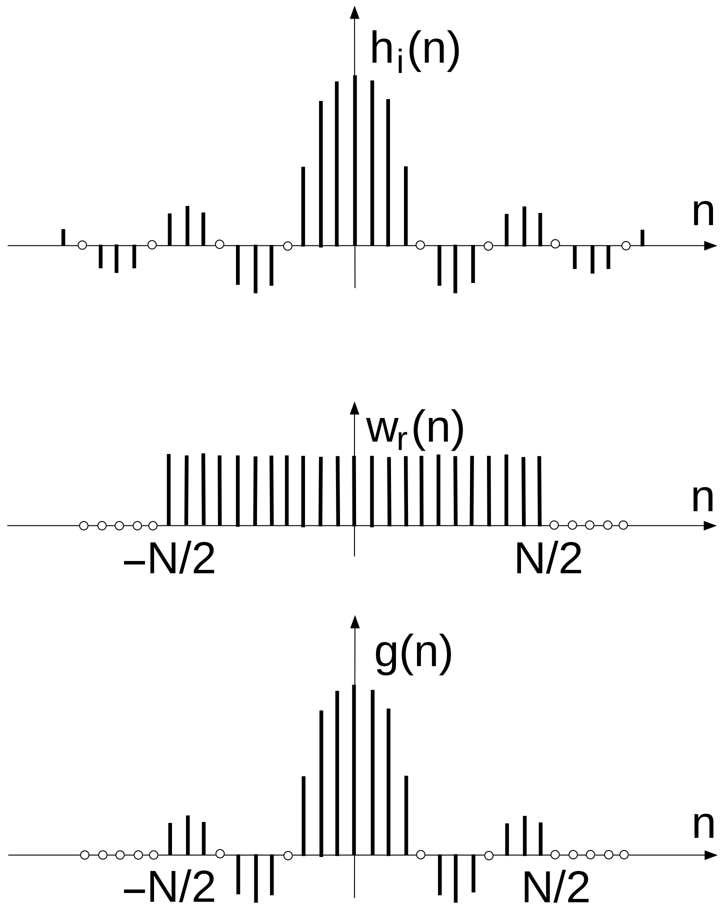
Recall that ideal lowpass filters are unstable.




$$h_i(n) = \frac{\sin \omega_c n}{\pi n}, \quad \sum_n |h_i(n)| = \infty$$

*Truncated impulse response*

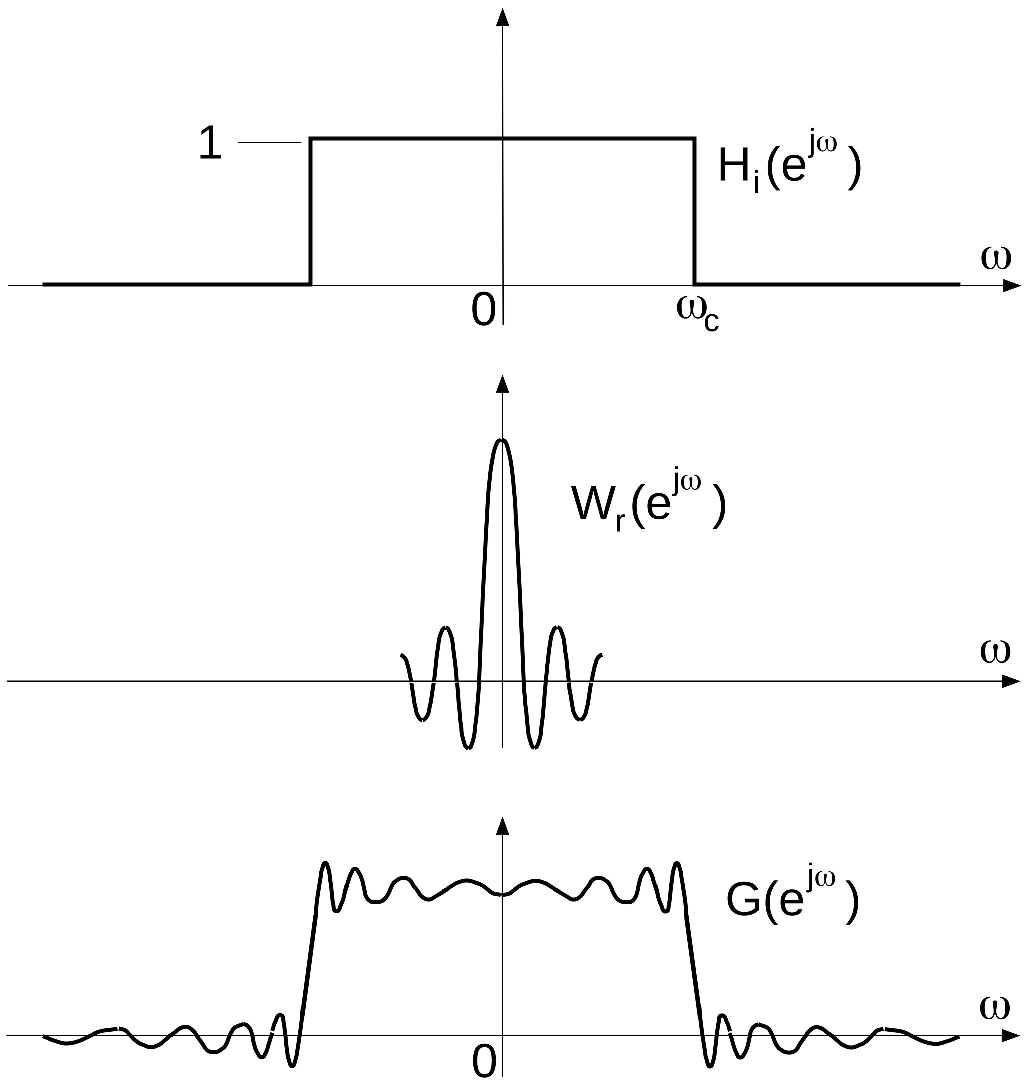



$g(n) = g(-n)$ so $g(n)$ is linear-phase FIR.

$$g(n) = h_i(n)w_r(n)$$

*Multiplication in time implies convolution in frequency.*

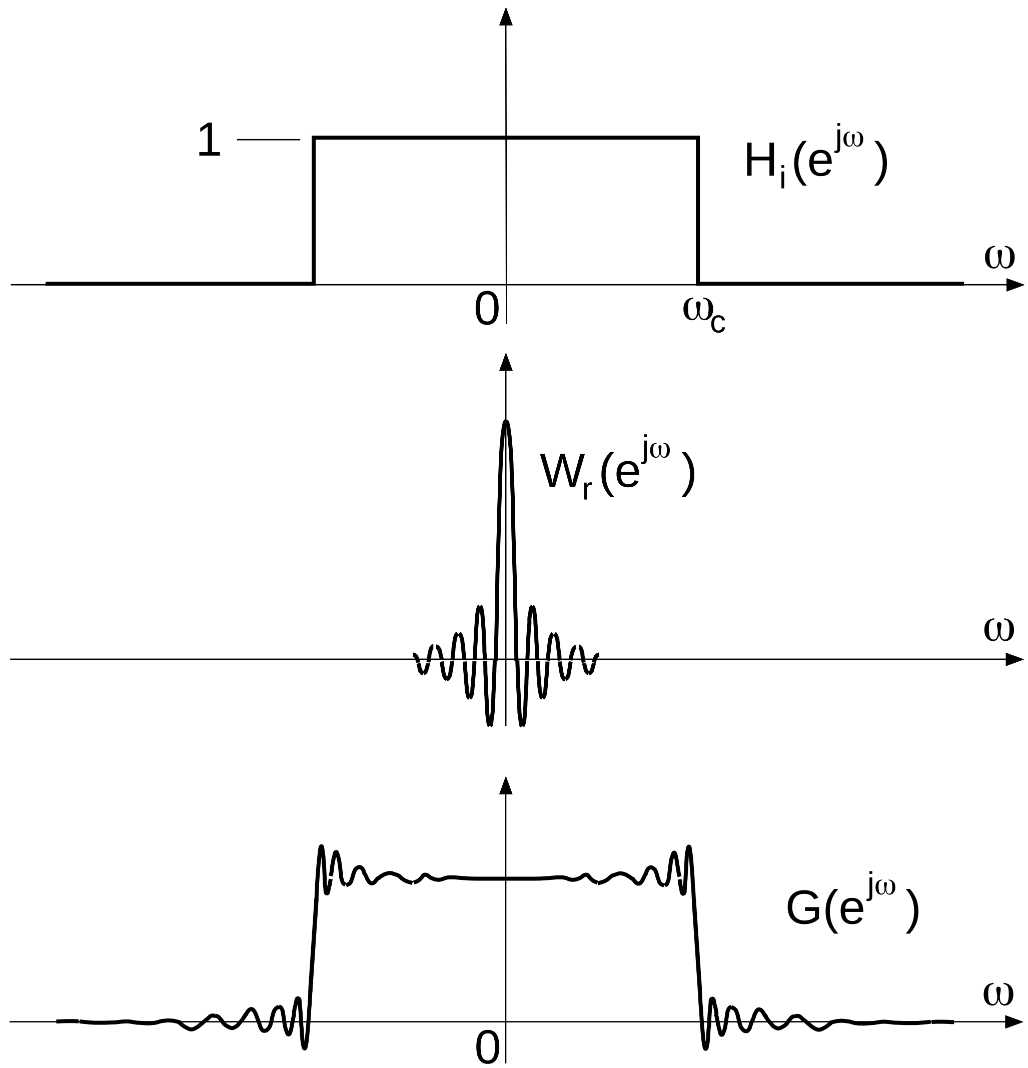
Convolution in frequency domain.




As $N$ increases, ripples get crowded but ripple size does not decrease.

*Gibbs phenomenon*

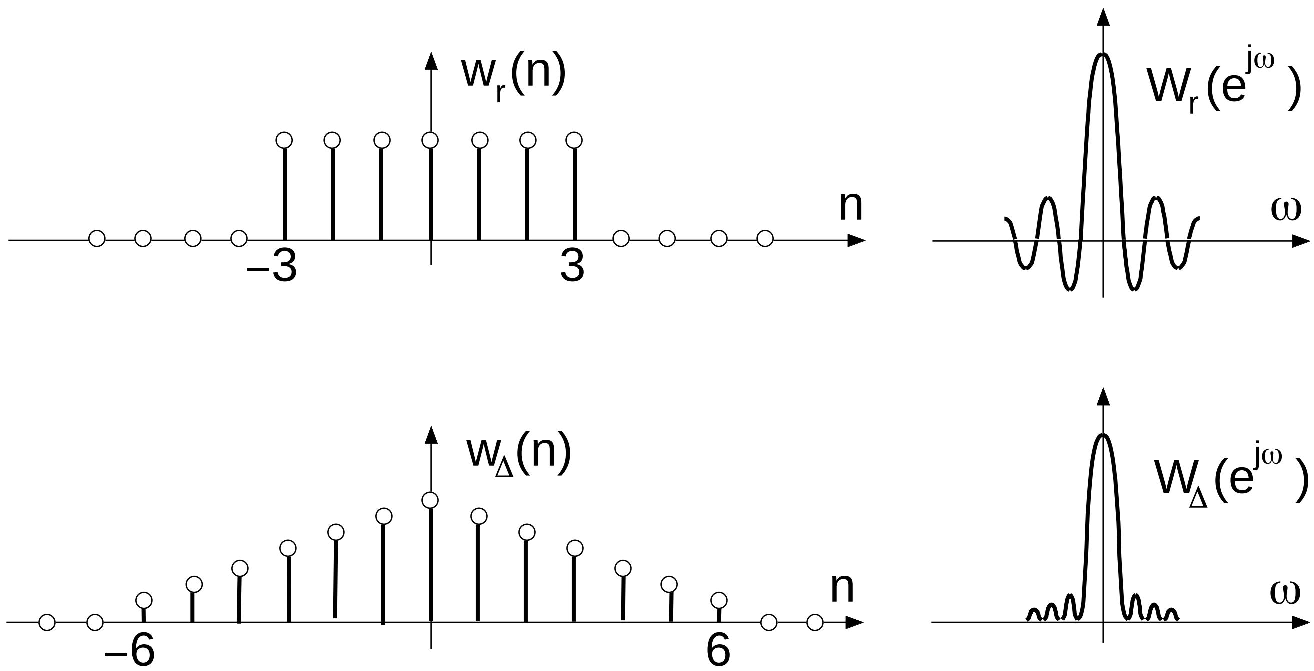
*Increasing the filter order N (window length)*




- Ripples get crowded near band edge
- Transition band gets narrower (sharper cutoff)
- But ripple size is about the same!

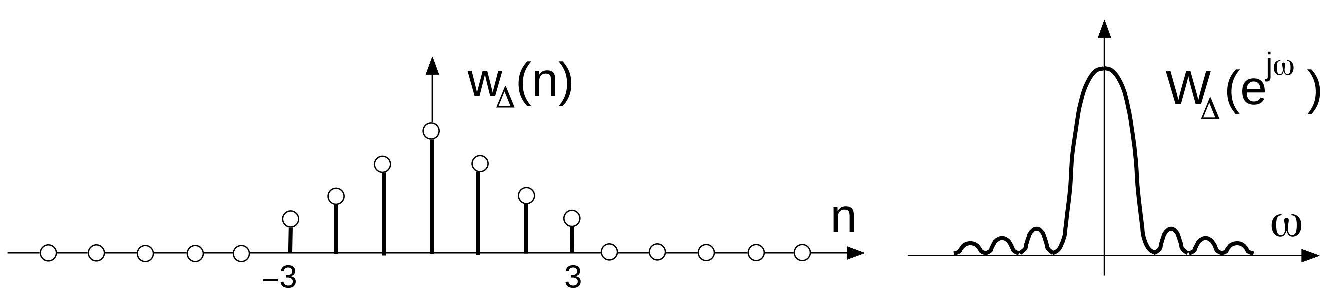
*How to reduce ripple size? Try better windows*!

*From rectangular to triangular window*

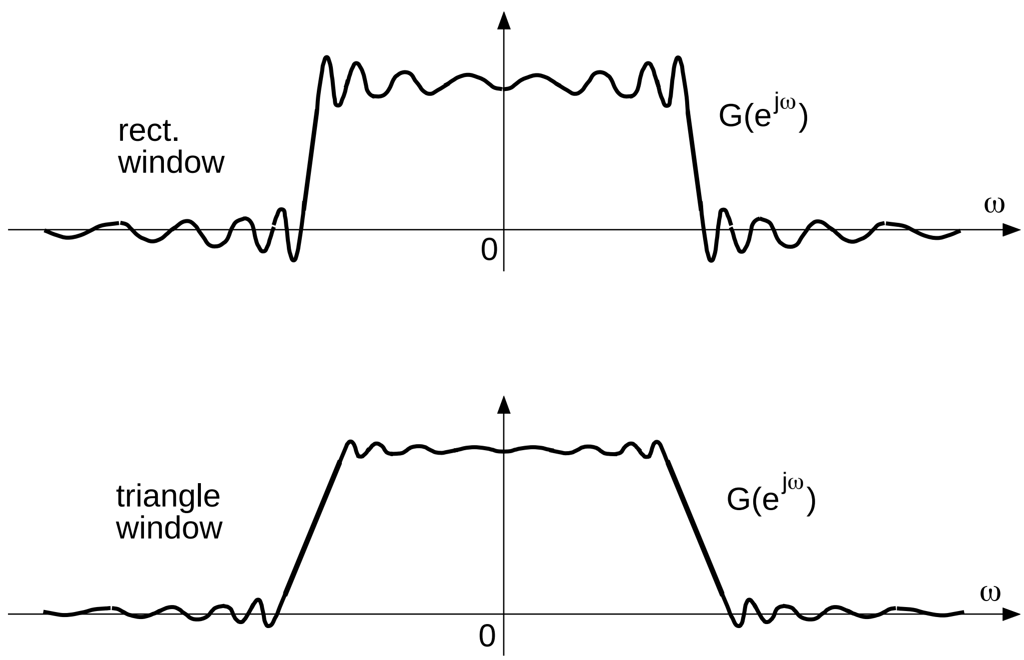



Ripples smaller but window is longer. Higher order filter.

*Tradeoff.* For fixed filter order, triangular window gives wider transition band and smaller ripple.



*Tradeoff.* For fixed filter order, triangular window gives smaller ripple at the cost of producing a wider transition band.

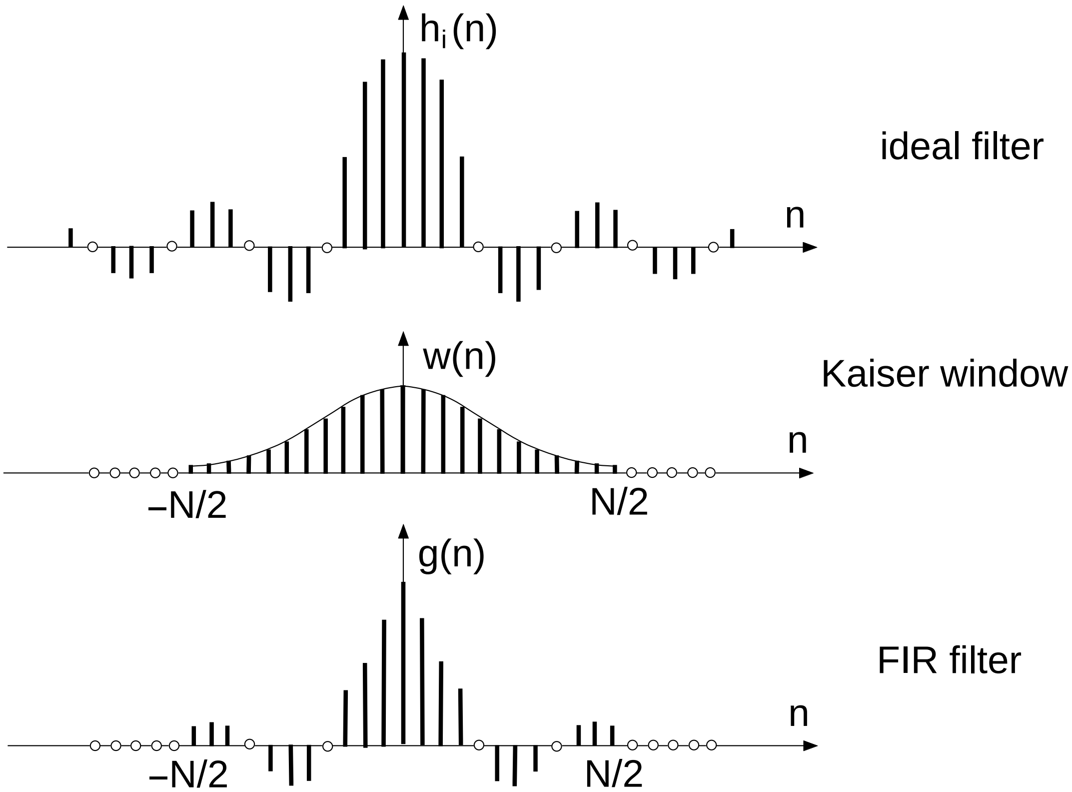



Is there a window that minimizes ripple size for a fixed transition bandwidth and fixed filter order.

Leads to the topic of:

*Optimal windows and optimal filters*

*Kaiser window: closed form approximation for an optimal window.*




$$w(n) = \begin{cases} I_0\left[\ \beta\sqrt{1-(n/0.5N)^2}\ \right]/I_0(\beta) & -N/2 \le n \le N/2 \\ 0 & \text{otherwise.} \end{cases}$$

$I_0(x) =$ modified zeroth-order Bessel function.

$$I_0(x) = 1 + \sum_{k=1}^{\infty}\left[(0.5\ x)^k/k!\right]^2, \quad \text{positive } \forall\ x$$

*Kaiser window of order $N$:*

$$w(n) = \begin{cases} I_0[\ \beta\sqrt{1-(n/0.5N)^2}\ ]/I_0(\beta) & -N/2 \leq n \leq N/2 \\ 0 & \text{otherwise} \end{cases}$$

*Choice of $\beta$ depends on stopband attenuation $A_s$:*

$$\beta = \begin{cases} 0.1102(A_s - 8.7) & A_s > 50 \\ 0.5842(A_s - 21)^{0.4} + 0.07886(A_s - 21) & 21 < A_s < 50 \\ 0.0 & A_s < 21 \end{cases}$$

*Choice of $N$ depends on $A_s$ and tansition BW* $\Delta f$:

$$N \approx \frac{A_s - 7.95}{14.36\Delta f}$$

- $N$ is proportional to $1/\Delta f$
- $N$ is proportional to $A_s$ for large $A_s$.
- $\delta_1 \approx \delta_2$. No separate control (for any window).

Given the specifications $\{\delta_1, \delta_2, \omega_p, \omega_s\}$, set

$$\Delta f = (\omega_s - \omega_p)/2\pi, \quad A_s = -20\log_{10}\delta$$

where $\delta$ is the smaller of $(\delta_1, \delta_2)$.