Transmultiplexers as precoders in modern digital communication: a tutorial review

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Abstract.[1] In this paper we review the recent impact of transmultiplexers in digital communications. Filter bank precoders, conditions for equalization, and multiuser interference cancellation are reviewed. The idea behind blind channel identification is reviewed as well. The emphasis is mostly on the theoretical infrastructure, and the list of references provide a wealth of related information.

I. INTRODUCTION

Transmultiplexers have been known in digital communications for many years. Historically the transmultiplexer has been viewed as a system that converts from time multiplexed components of a signal to a frequency multiplexed version, and back [1], [2], [18]. The mathematical theory of transmultiplexers however allows more general interpretations and, therefore, applications. Some of these include channel equalization, channel identification and so forth. The role of transmultiplexers in digital communications has gained new importance because of many recent results in filter bank precoders. The works of Giannakis et al., Xia, and Lin and Phoong have had a particular impact in this area. The pioneering work of Giannakis et al. shows that such precoders not only allow equalization of linear finite-spread channels, but also blind equalization based only on second order statistics. Furthermore, cancellation of multiuser interference in CDMA channels without knowledge of channel coefficients has been shown to be possible.

In this paper we review the fundamental infrastructure behind these methods. In Sec. II, the transmultiplexer system is introduced and the mathematical framework deveoped. In Sec. III we present the theory behind the cancellation of multiuser interference. To the best of our knowledge the approach used here is new. It gives rise to several well known systems as special cases, including the single user DMT system [16], [8], [21], and the multiuser Amour system [4]. Section IV describes the meaning of bandwidth expansion and then derives the Amour system of [4] which was introduced in the literature in a different context, namely, the cancellation of multiuser interference. In Sec. V we make brief remarks on handling the effects of channel noise. In Sec. VI the fundamentals behind blind identification of channels using transmultiplexers is reviewed. Some of the important topics that are not covered in detail are briefly mentioned in Sec. VII. Important references are cited for further reading, and there are excellent papers in this special session covering related topics.

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Notations. Standard multirate notations from [18] will be freely used. For example the notation $[H(z)]_{\downarrow M}$ represents the z-transform of the decimated version $h(Mn)$. Lower and upper case notations such as $h_k(n)$ and $H_k(z)$ will be consistently used to denote sequences and their z-transforms. The "communications notations" used here are quite standard, though some readers might benefit by reading [20] for this.

II. THE TRANSMULTIPLEXER SYSTEM

Most of our discussions in this paper will center around the structure shown in Fig. 1 (and its generalization in Fig. 2) called a transmultiplexer. The signals $s_k(n)$ are symbol streams (such as PAM or QAM signals, [11], [20]). These could be symbols generated by different users who wish to transmit messages over the channel. Or they could be different independent parts of the signals generated by one user [20]. The distinction between these two scenarios is not required for our discussions here. The symbol streams $s_k(n)$ are passed through the interpolation filters or transmitter filters $F_k(z)$ to produce the signals

$$x_k(n) = \sum_i s_k(i) f_k(n - iP)$$

The filters $F_k(z)$ are also called pulse shaping filters because they take each sample of $s_k(n)$ and "put a pulse $f_k(n)$ around it". The sum $x(n)$ of the signals $x_k(n)$ is then transmitted over a common channel. The channel is described by a linear time invariant filter $C(z)$ followed by additive noise. At the receiver end, the filters $H_k(z)$ have the task of separating the signals and reducing them to the original rates by P-fold decimation.

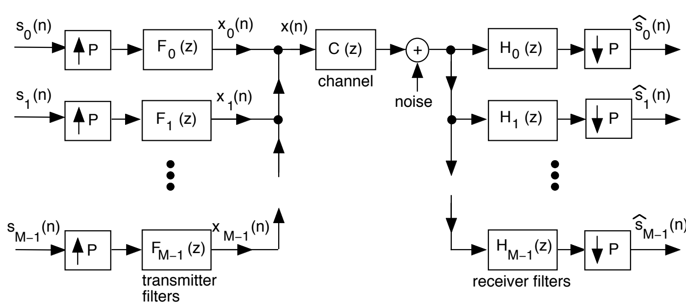

Fig. 1. The M-user transmultiplexer.

II.1. Preliminary remarks

Notice that since the M signals are mutiplexed into one channel, it is necessary to have $P \geq M$. When $P > M$ we have a *redundant transmultiplexer*, whereas $P = M$ corresponds to a *minimal* transmultiplexer.

The received signals $\widehat{s}_k(n)$ in general are different from $s_k(n)$ for several reasons. First, there is multiuser interference or **MUI**. This means that $\widehat{s}_k(n)$ is affected not only by $s_k(n)$ but also by $s_m(n), m \neq k$. Second,the channel $C(z)$ introduces a linear distortion called intersymbol interference or **ISI** (even when $M = 1$), and finally there is additive noise. The task at the receiver is therefore to minimize the effects of these distortions so that the transmitted symbols $s_k(n)$ can be detected from $\widehat{s}_k(n)$ with acceptably low probabilities of error. In absence of noise it is possible to compensate or **equalize** the effect of the channel completely and obtain perfect symbol reconstruction, that is $\widehat{s}_k(n) = s_k(n)$. The theory of perfect-reconstruction transmultiplexers for the case $P = M$ was developed in [22] and [6]. The case $P > M$ is more useful, as it eliminates some of the difficulties associated with practical filter design. For example when $P > M$ it is possible to equalize an FIR channel with the help of FIR filters $H_k(z), F_m(z)$ alone [8,13].

A special case of the redundant transmultiplexer has been widely used in DSL systems which use the discrete multitone modulation or **DMT** techniques in the transceiver [5], [8], [20]. In this system the filters are chosen from a simple uniform DFT filter bank (this appears at the end of this section). The more general theory of redundant transmultiplexers is fairly recent. Noteworthy here are the fundamental contributions from Lin and Phoong and early contributions from Xia (see references at the end). Pioneering developments on redundant transmultiplexers, also called *filter bank precoders* came from the group of Giannakis, et al., who showed both equalizability and blind identifiability (these terms will become clear as we proceed further).

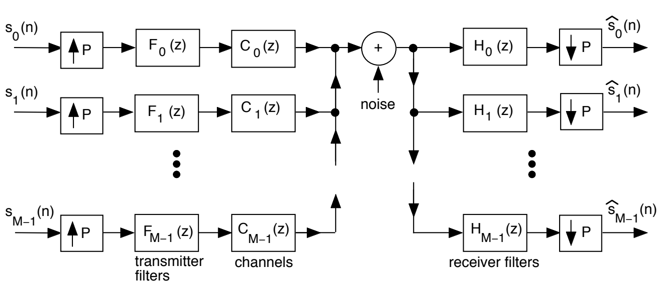

Fig. 2. Generalization of the M-user transmultiplexer.

A generalization of the transmultiplexer system is shown in Fig. 2. Again there are M users transmitting the symbol streams $s_m(n)$, but over M *different channels* $C_m(z)$. A noisy superimposition of these is received by the M receivers, which are expected to separate out the components $s_m(n)$. If the channel transfer functions $C_m(z)$ are identical for all m this reduces to the traditional transmultiplexer of Fig. 1. This generalization has become important in the context of multiuser communications over wireless channels. A technique for cancellation of MUI in these systems was first developed by Giannakis, et al. [4].

II.2. Mathematical analysis

Even though Fig. 2 is more general than Fig. 1, it is just as easy to analyze. So we will start with this figure. An important result in the theory of multirate systems is called the polyphase identity [18]. This states that if a linear time invariant (LTI) filter $G(z)$ is sandwiched between an expander and decimator as shown in Fig. 3 then the result is equivalent to an LTI system with impulse response $d(n) = g(Pn)$ (decimated version). That is, the transfer function of the overall system is $D(z) = [G(z)]_{\downarrow P}$. Returning to Fig. 2 the transfer function from $s_m(n)$ to $\widehat{s}_k(n)$ is therefore given by

$$T_{km}(z) = [H_k(z)C_m(z)F_m(z)]_{\downarrow P} \tag{1}$$

If we choose the filters $\{F_m(z)\}$ and $\{H_k(z)\}$ such that

$$[H_k(z)C_m(z)F_m(z)]_{\downarrow P} = \delta(k-m), \tag{2}$$

then multiuser interference is cancelled and there is perfect recovery of symbols, i.e., the **PR property**.

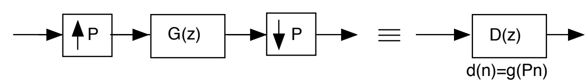

Fig. 3. The polyphase identity.

It is possible to satisfy (2) even when $P = M$. Readers familiar with the idea of *biorthogonal filter banks* will recognize that with $G_m(z) \stackrel{\Delta}{=} C_m(z)F_m(z)$, the sets $\{H_k(z)\}$ and $\{G_m(z)\}$ satisfying (2) form a biorthogonal filter bank. However, even when the channels $C_m(z)$ are known and FIR, it is not true that there exist FIR solutions or stable IIR solutions $F_m(z), H_k(z)$ satisfying (2). For example when $P = M = 1$, the condition is like $H(z)C(z)F(z) = 1$ and cannot be satisfied when all three transfer functions are FIR.

If we allow an interpolation factor $P > M$ (redundant transmultiplexer) then there are many advantages. For example assume the channels are FIR:

$$C_m(z) = \sum_{n=0}^{L} c_m(n) z^{-n} \tag{3}$$

Then if we choose $P \geq M + L$, there is a clever way to obtain perfect symbol recovery with FIR filters $F_m(z)$ and $H_k(z)$, as we shall see.

II.3. Eliminating interblock interference

Figure 4 shows the mth transmitter and kth receiver redrawn in polyphase form [18]

$$F_m(z) = \sum_{i=0}^{P-1} z^{-i} R_{i,m}(z^P), \ H_k(z) = \sum_{i=0}^{P-1} z^{i} E_{k,i}(z^P) \tag{4}$$

With noise ignored, the path between the mth transmitter and kth receiver can be described in terms of a matrix $\mathbf{C}_m(z)$ whose elements are

$$[\mathbf{C}_m(z)]_{\ell,i} = [z^{\ell-i}C_m(z)]_{\downarrow P}$$

For example when $L = 2$ and $P = 5$, $\mathbf{C}_m(z)$ is

$$\begin{array}{ccccc} & P-L & & | & L \\ \end{array}$$
$$\begin{pmatrix} c_m(0) & 0 & 0 & | & z^{-1}c_m(2) & z^{-1}c_m(1) \\ c_m(1) & c_m(0) & 0 & | & 0 & z^{-1}c_m(2) \\ c_m(2) & c_m(1) & c_m(0) & | & 0 & 0 \\ 0 & c_m(2) & c_m(1) & | & c_m(0) & 0 \\ 0 & 0 & c_m(2) & | & c_m(1) & c_m(0) \end{pmatrix}$$

This is called the **blocked version** of $C_m(z)$.

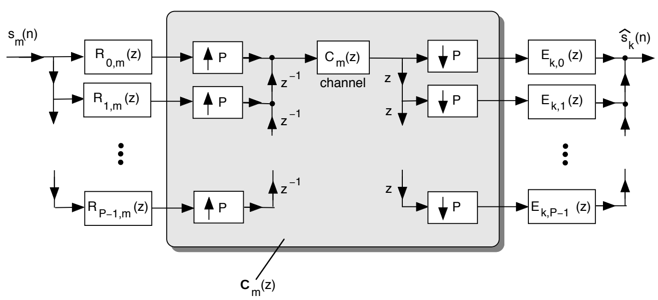

Fig. 4. The mth transmitter and kth receiver in polyphase form.

The preceding matrix is called a *pseudocirculant* and the interested reader can find out more about it in [18] and references therein. The first $P-L$ columns have constant entries and the last L columns have z^{-1} in them. Similarly the last $P-L$ rows have constant entries. Thus the blocked version can be partitioned in two ways:

$$\mathbf{C}_m(z) = P \overset{\quad P-L \qquad L}{\begin{pmatrix} \mathbf{A}_m & \mathbf{B}(z) \end{pmatrix}} = \begin{matrix} L \\ P-L \end{matrix} \overset{P}{\begin{pmatrix} \mathbf{C}(z) \\ \mathbf{D} \end{pmatrix}} \qquad (5)$$

where $\mathbf{A}_m$ is a $P \times (P-L)$ matrix and $\mathbf{D}$ is a $(P-L) \times P$ matrix (both constant Toeplitz matrices). In Fig. 4 the transfer function from $s_m(n)$ to $\widehat{s}_k(n)$ is given by

$$[\, E_{k,0}(z) \quad E_{k,1}(z) \quad \ldots \quad E_{k,P-1}(z)\,]\, \mathbf{C}_m(z) \begin{bmatrix} R_{0,m}(z) \\ R_{1,m}(z) \\ \vdots \\ R_{P-1,m}(z) \end{bmatrix}$$

Using (5) this can be written in either one of the two forms

$$[\, E_{k,0}(z)\ E_{k,1}(z) \ldots E_{k,P-1}(z)\,] \begin{pmatrix} \mathbf{A}_m & \mathbf{B}(z) \end{pmatrix} \begin{bmatrix} R_{0,m}(z) \\ R_{1,m}(z) \\ \vdots \\ R_{P-1,m}(z) \end{bmatrix}$$

or

$$[\, E_{k,0}(z) \quad E_{k,1}(z) \ldots E_{k,P-1}(z)\,] \begin{pmatrix} \mathbf{C}(z) \\ \mathbf{D} \end{pmatrix} \begin{bmatrix} R_{0,m}(z) \\ R_{1,m}(z) \\ \vdots \\ R_{P-1,m}(z) \end{bmatrix}$$

Since $\mathbf{B}(z)$ and $\mathbf{C}(z)$ have z^{-1} in them, they represent interference between input vectors occuring at different times at the input of $\mathbf{C}_m(z)$. This is called inter-block interference or **IBI**, and it is convenient to eliminate it. We will describe two ways to construct the transmitter and receiver filters in such a way that IBI is eliminated.

1. ***Zero-padding***. In the first method we eliminate $\mathbf{B}(z)$ by setting

   $$R_{P-L,m}(z) = \ldots = R_{P-1,m}(z) = 0.$$

   That is, we insert a block of L zeros at the end of each block of $P-L$ symbols (Fig. 5(a)). This zero-padding method is also called *zero-prefixing*.

2. ***Zero-jamming***. In the second method we eliminate $\mathbf{C}(z)$ by choosing

   $$E_{k,0}(z) = \ldots = E_{k,L-1}(z) = 0.$$

   That is, we replace a block of L samples with zeros, at the beginning of each block of P successive *received* symbols (Fig. 5(b)).

Our discussions in this paper will be restricted to the zero padding case. In the zero-padding method the transfer function $T_{km}(z)$ from $s_m(n)$ to $\widehat{s}_k(n)$ is given by

$$[\, E_{k,0}(z)\ E_{k,1}(z) \ \ldots \quad E_{k,P-1}(z)\,]\, \mathbf{A}_m \begin{bmatrix} R_{0,m}(z) \\ R_{1,m}(z) \\ \vdots \\ R_{P-L-1,m}(z) \end{bmatrix} \qquad (6)$$

where the $P \times (P-L)$ matrix $\mathbf{A}_m$ has the form

$$\mathbf{A}_m = \begin{bmatrix} c_m(0) & 0 & \ldots & 0 \\ c_m(1) & c_m(0) & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ c_m(L) & & & \\ 0 & c_m(L) & & \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \ldots & c_m(L) \end{bmatrix} \qquad (7)$$

This represents the effects of the mth channel completely.

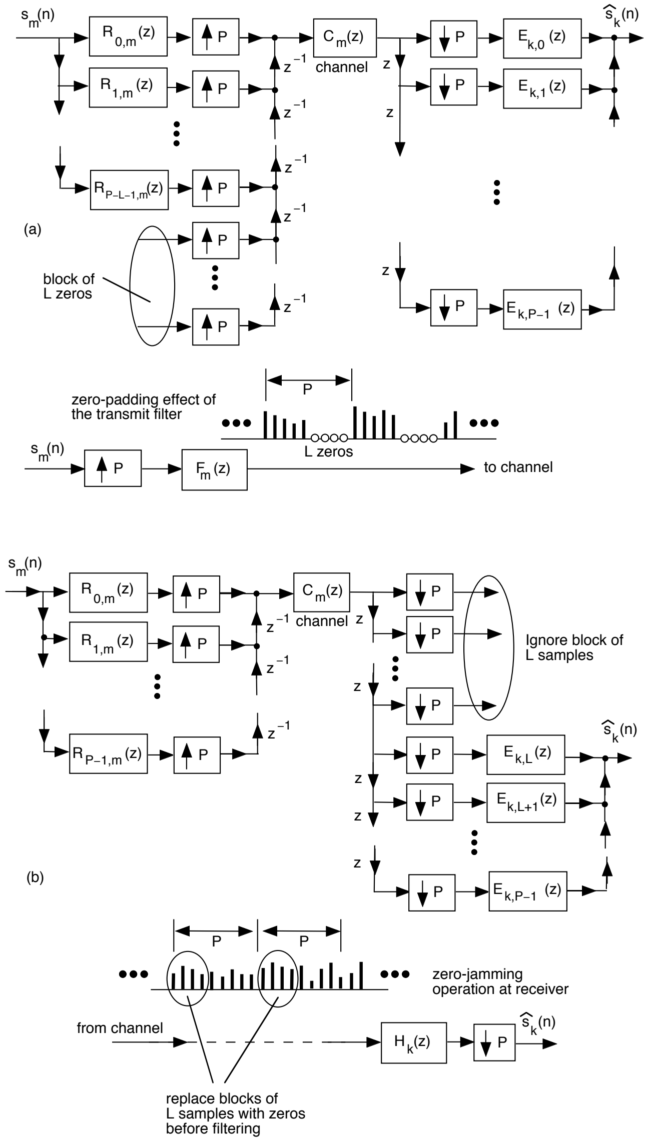

Fig. 5. Two ways to force the effective channel matrix to be a constant. (a) Zero-padding, and (b) zero-jamming.

III. CANCELLING MULTIUSER INTERFERENCE

The matrix $\mathbf{A}_m$ given above is a *full-banded Toeplitz matrix*. Because of this it has a very beautiful property: given any nonzero number ρ_k, the matrix satisfies the equation

$$[\,1\;\rho_k^{-1}\;\ldots\;\rho_k^{-(P-1)}\,]\,\mathbf{A}_m = C_m(\rho_k)\,[\,1\;\rho_k^{-1}\;\ldots\;\rho_k^{-(P-L-1)}\,] \tag{8}$$

where $C_m(z) = \sum_n c_m(n)z^{-n}$ as usual. This is quite readilty verified directly.[2] We exploit this in the choice of our receiver and transmitter filters, and show how MUI can be cancelled. Consider the transfer function $T_{km}(z)$ given by (6). Suppose we choose

$$H_k(z) = a_k(1+\rho_k^{-1}z+\rho_k^{-2}z^2+\ldots+\rho_k^{-(P-1)}z^{(P-1)}) \tag{9}$$

$$F_m(z) = r_{0,m}+r_{1,m}z^{-1}+\ldots+r_{P-L-1,m}z^{-(P-L-1)} \tag{10}$$

where ρ_k are distinct for $0 \le k \le M-1$. Using the identity (8), Eq. (6) representing $T_{km}(z)$ can be rearranged as

$$\begin{aligned} & a_k\,[\,1\;\rho_k^{-1}\;\ldots\;\rho_k^{-(P-1)}\,]\,\mathbf{A}_m \begin{bmatrix} r_{0,m} \\ r_{1,m} \\ \vdots \\ r_{P-L-1,m} \end{bmatrix} \\ = \; & a_k C_m(\rho_k)\,[\,1\;\rho_k^{-1}\;\ldots\;\rho_k^{-(P-L-1)}\,] \begin{bmatrix} r_{0,m} \\ r_{1,m} \\ \vdots \\ r_{P-L-1,m} \end{bmatrix} \\ = \; & a_k C_m(\rho_k) F_m(\rho_k) \end{aligned}$$

Thus the transfer function from $s_m(n)$ to $\widehat{s}_k(n)$ is

$$T_{km}(z) = a_k C_m(\rho_k) F_m(\rho_k) \tag{11}$$

which is a constant independent of z. Assume further that the multipliers a_k are chosen as

$$a_k = \frac{1}{C_k(\rho_k)} \tag{12}$$

Then the perfect symbol recovery condition $T_{km}(z) = \delta(k-m)$ becomes

$$F_m(\rho_k) = \delta(k-m), \quad 0 \le k,m \le M-1 \tag{13}$$

Even with ρ_k chosen as arbitrary (but distinct) numbers, these can be satisfied as long as $F_m(z)$ have M degrees of freedom (FIR with order $\ge M-1$). This gives the condition $P \ge M+L$ which we shall replace with

$$P = M+L \tag{14}$$

Then the preceding conclusions are valid as long as the channel order $\le L$. The multipliers a_k in Eq. (12) which are part of the receiver filters (9) can be regarded as z-domain equalizers. If $\rho_k = e^{j\omega_k}$ these become *frequency domain equalizers*.

A number of conclusions can be made here. The condition (13) guaratees that multiuser interference is cancelled *even if the channels are unknown.* Only their order L needs to be known. The knowledge of the channels is required only to design the equalizers (12) for each receiver. Since FIR channels have only finite number of zeros, we can always choose ρ_k so that $C_k(\rho_k) \neq 0$. *So, channel equalization is always possible*!

[2]Since Toeplitz matrices represent convolution (LTI filtering) when appropriately used, this property can be regarded as a manifestation of the elementary fact that LTI systems reproduce exponentials [10].

Example. *The DFT filter bank.* Assume $C_m(z) = C(z)$ for all m, $P = M+L$, and $\rho_k = W^{-k},\ W \stackrel{\Delta}{=} e^{-j2\pi/M}$. Then

$$H_k(z) = \frac{1}{C(W^{-k})} \sum_{n=0}^{P-1} z^n W^{nk} \tag{15}$$

The PR condition (13) yields the transmit filters $F_m(z) = \sum_{n=0}^{M-1} W^{-mn} z^{-n}/M$.

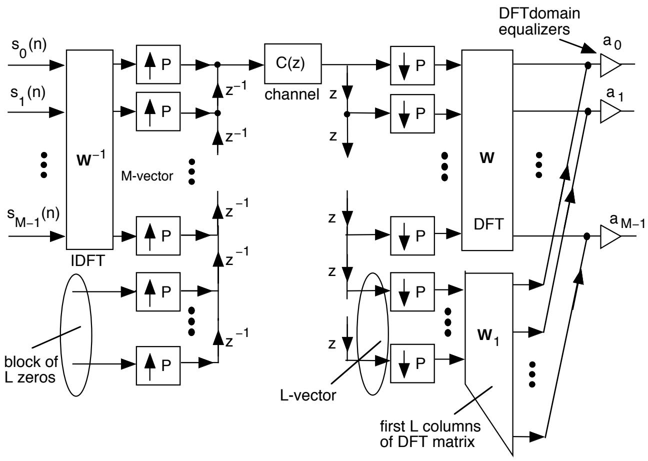

Fig. 6. Zero-padding at the transmitter and cyclic-prefix-like equalizers at the receiver.

If all users are in one place[3] the system can be drawn as shown in Fig. 6 where $\mathbf{W}$ is the $M \times M$ DFT matrix and $\mathbf{W}_1$ is the submatrix of $\mathbf{W}$ obtained by retaining the first $L = (P - M)$ columns. The equalizers

$$a_k = 1/C(W^{-k}) = 1/C(e^{j2\pi k/M}) \tag{16}$$

invert the channel frequency response sampled on the DFT grid.

If we start with the *zero-jamming* approach instead of zero-padding, we can develop a similar example. After going through the details we will discover the transceiver shown in Fig. 7. Readers familiar with the **cyclic prefix** system [16] will readily recognize it in this figure! Thus the cyclic prefix system is a special case of the equalizer system developed in this section and therefore has the perfect symbol recovery property in absence of noise. Our analysis above shows that the equalizers a_k appearing in cyclic prefix systems also appear naturally in a *zero padding system.*

[3]In the multiuser scenario, the symbol streams $s_m(n)$ are separate independent users. On the other hand, in a context like the DMT channel, $s_m(n)$ are obtained from one stream $s(n)$ by parsing [20].

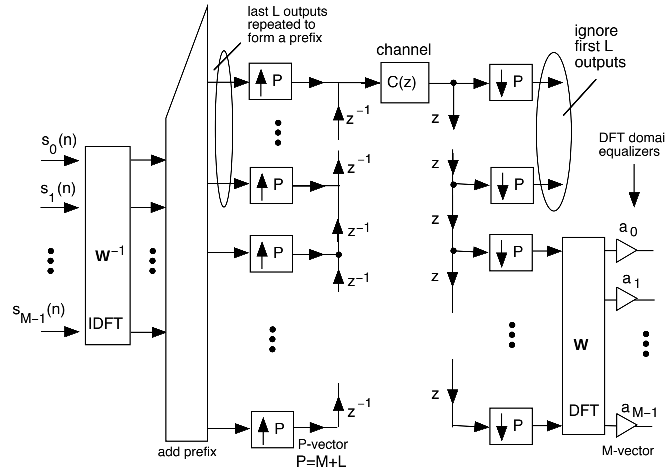

Fig. 7. The cyclic prefix system used in discrete multitone modulation.

IV. CONTROLLING BANDWIDTH EFFICIENCY

The transmitter system of Fig. 2 makes a linear combination of M independent symbol streams. If these have to be separated successfully later, then the spacing between the samples of $s_k(n)$ must be at least M times larger than the spacing T between the symbols entering the channels $C_k(z)$. But the actual spacing between samples of $s_k(n)$ is PT secs. See Fig. 8(a) which demonstrates the idea with $T = 1$. The excess space, measured by the ratio

$$\gamma = \frac{P}{M} = \frac{M+L}{M},$$

is called the *bandwidth expansion factor.* It is a price paid, in terms of the redundancy, which allows equalization of FIR channels of order L.

A slight variation of the redundant transmultiplexer system of Fig. 2 results if we make each user look like K users. This can be done by blocking the mth user K-fold as in Fig. 9. The K "subusers" $s_{m,i}(n)$ are merely substreams of $s_m(n)$. The system is mathematically equivalent to MK users, though the MK channels are not all different. The perfect recovery requirement is now $\widehat{P} \geq MK + L$ which we replace with

$$\widehat{P} = MK + L \tag{17}$$

The advantage of this system is that the banwidth expansion factor is now

$$\widehat{\gamma} = \frac{\widehat{P}}{MK} = \frac{MK+L}{MK} \tag{18}$$

which can be made arbitrarily close to unity by making K large. See Fig. 8(b). MUI cancellation and equalization can be achieved exactly as in the previous section by appropriate design of the transmit and receiver filters. This is precisely the **Amour system** developed by Giannakis et al. [4] for MUI cancellation in CDMA systems wherein, each of the $F_{m,i}(z)$ is regarded as a CDMA code (each user has

K codes). There are K different coefficients ρ_k used by each user, and these are called signature points in [4].

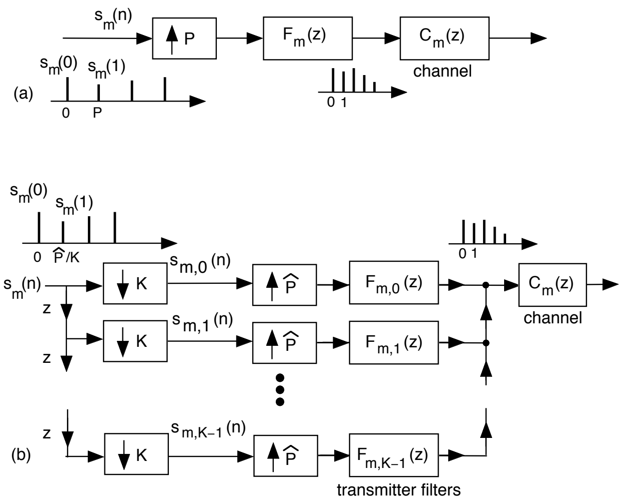

Fig. 8. Explaining bandwidth expansion.

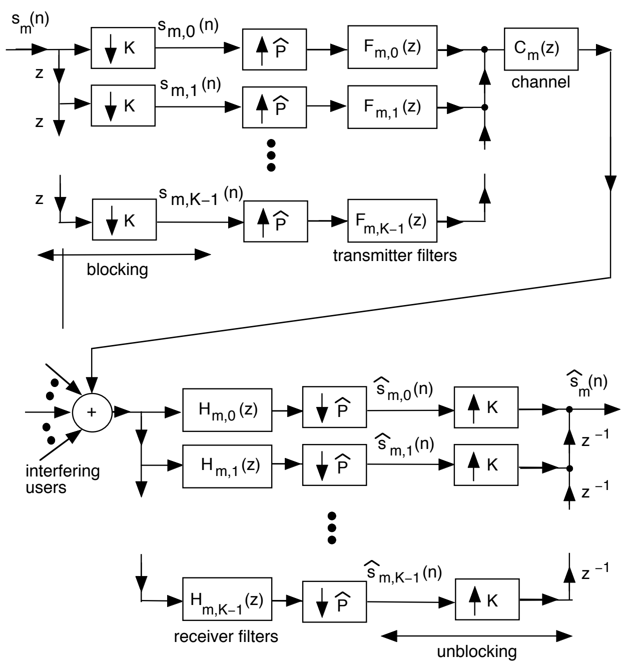

Fig. 9. The mth user viewed as K separate users.

V. OPTIMIZING FOR NOISE

If the statistics of the channel noise is known then it is possible to optimize the receiver to reduce its effect. To give a flavor for this we describe a scenario developed by Lin and Phoong [8]. Consider again the transmultiplexer of Fig. 1. With the transmitter and receiver filter banks expressed in polyphase matrix form [18] we can draw the transceiver as in Fig. 10. Assume as before that the filters have order $< P$ so $\mathbf{E}(z)$ and $\mathbf{R}(z)$ are constants. In the zero-padding scheme

$$\mathbf{R}(z) = \begin{bmatrix} \mathbf{R}_0 \\ \mathbf{0} \end{bmatrix} \tag{19}$$

where $\mathbf{R}_0$ is $M \times M$. The transfer function from $\mathbf{s}(n)$ to $\widehat{\mathbf{s}}(n)$ is the product $\mathbf{T}(z) = \mathbf{E}(z)\mathbf{C}_b(z)\mathbf{R}(z)$. where $\mathbf{C}_b(z)$ is the blocked version of the channel (like $\mathbf{C}_m(z)$ in Fig. 4). With $\mathbf{R}(z)$ restricted as above, this becomes $\mathbf{EAR}_0$, where $\mathbf{A}$ is as in Eq. (7). The condition for perfect recovery in absence of noise is therefore

$$\mathbf{EAR}_0 = \mathbf{I} \tag{20}$$

With $\mathbf{R}_0$ and $\mathbf{A}$ given, the $M \times P$ matrix $\mathbf{E}$ has to be chosen as a left inverse of $\mathbf{AR}_0$. Since $P > M$ the left inverse is not unique. The left inverse which minimizes the channel noise at the receiver input is worked out in [8].

Consider again Fig. 1 and assume no redundancy, that is, $P = M$. If we assume that we are allowed to have $1/C(e^{j\omega})$ at the receiver and restrict the filter bank $\{F_m(z), H_k(z)\}$ to the class of orthonormal filter banks [18], then an interesting result can be proved. Namely the optimum filter bank which minimizes the probability of error (for fixed bit rate and transmitted power) is the so-called principal component filter bank or **PCFB** for the effective power spectrum

$$S_{eff}(e^{j\omega}) \stackrel{\Delta}{=} S_{ee}(e^{j\omega})/|C(e^{j\omega})|^2,$$

where $S_{ee}(e^{j\omega})$ is the power spectrum of the additive noise $e(n)$. Further details on this result can be found in [21].

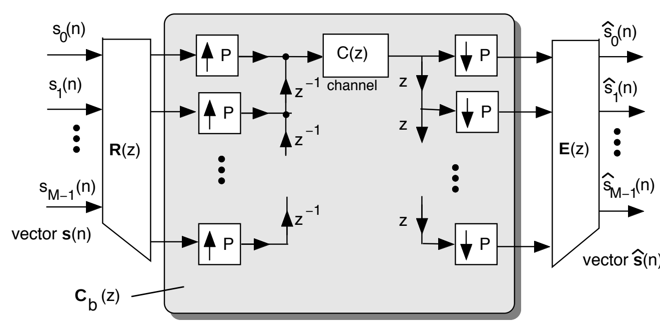

Fig. 10. Transmultiplexer of Fig. 1 in polyphase form.

All systems described in the preceding paragraphs fall under the category of *zero-forcing equalizers*. i.e., equalizers which tend to cancel the effect of $C(z)$ completely without taking into account the effects of noise. It turns out that this is not necessarily the best thing to do (*intuition*: if $C(e^{j\omega})$ is very small for some frequencies, then $1/C(e^{j\omega})$ can amplify the channel noise severely). There exist equalizers based on minimizing the mean squared error between $s_k(n)$ and $\widehat{s}_k(n)$. These are called MMSE equalizers. Some details can be found in [13] and [4].

VI. BLIND IDENTIFICATION

In many practical situations as in mobile communications, the channel transfer functions are unknown, and have to be estimated before they can be equalized. Such estimation can be done either with the help of training signals or by blind identification methods. It is well known that with non redundant systems ($P = M$) blind identification is not possible unless we use fourth order moments such as the Kurtosis of the data [15], [17]. An important result in recent years is the observation that blind identification is indeed possible without the use of fourth order moments, if we use *redundant* transmultiplexers or filter bank precoders [3], [14].

Consider again the single channel system of Fig. 10. Assume as in Sec. V that the channel is FIR with order $\leq L$, that the receiver filters have order $\leq P-1$, and that the transmitting filters have order $\leq M-1$. In particular therefore $\mathbf{R}(z)$ is as in Eq. (19). Figure 11 shows the path from the transmitted symbols to the channel output $y(n)$. For convenience we consider the blocked version $\mathbf{y}(n)$ as indicated. With the vector $\mathbf{s}(n)$ as defined in the figure, we then have

$$\mathbf{y}(n) = \mathbf{A}\mathbf{R}_1\mathbf{s}(n)$$

where $\mathbf{A}$ is as in (7) and reproduced below:

$$\mathbf{A} = \begin{bmatrix} c(0) & 0 & \dots & 0 \\ c(1) & c(0) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ c(L) & & & \\ 0 & c(L) & & \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \dots & c(L) \end{bmatrix}$$

This is a full-banded Toeplitz matrix representing the FIR channel of order $\leq L$.

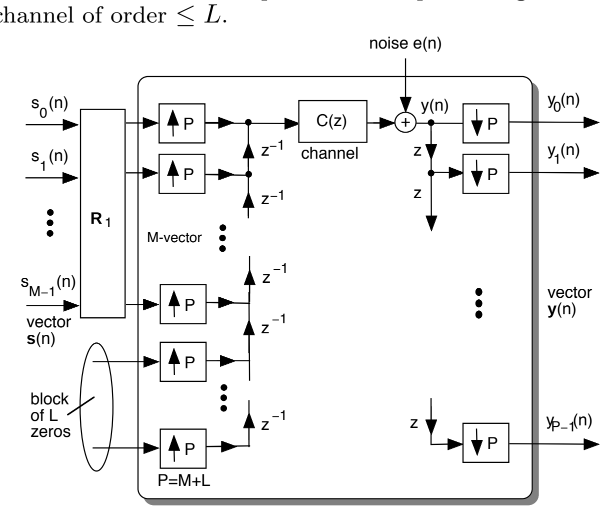

Fig. 11. The zero padding system with precoder $\mathbf{R}_1$.

Assume the channel $c(n)$ is unknown. We now argue that the observation of $\mathbf{y}(n)$ can be used to identify the channel $c(n)$ upto a scale-factor ambiguity. This is called *blind identification* because the input stream $\mathbf{s}(n)$ is unknown, unlike in training-based channel identification. Briefly, imagine we observe the output vector $\mathbf{y}(n)$ for a certain duration, say $0 \leq n \leq J-1$, and write the equation

$$\underbrace{\left[\,\mathbf{y}(0) \quad \mathbf{y}(1) \quad \dots \quad \mathbf{y}(J-1)\,\right]}_{\mathbf{Y}\text{ matrix; size } P\times J}$$

$$= \underbrace{\mathbf{A}}_{P\times M} \underbrace{\mathbf{R}_1}_{M\times M} \underbrace{\left[\,\mathbf{s}(0) \quad \mathbf{s}(1) \quad \dots \quad \mathbf{s}(J-1)\,\right]}_{\mathbf{S}\text{ matrix; size } M\times J} \qquad (21)$$

At this point we assume that the symbol stream $\mathbf{s}(n)$ is *rich,* that is, there exists a J such that $\mathbf{S}$ has full rank M. Since $\mathbf{A}$ and $\mathbf{R}_1$ have rank M, the product on the right hand side of Eq. (21) has rank M. So the $P \times J$ data matrix $\mathbf{Y}$ has rank M, and there are $P-M$ or L linearly independent vectors orthogonal to all the columns in $\mathbf{Y}$. That is, there is a $L \times P$ matrix $\mathbf{V}$ with L independent rows such that

$$\mathbf{V}\mathbf{Y} = \mathbf{V}\mathbf{A}\mathbf{R}_1\mathbf{S} = \mathbf{0} \qquad (22)$$

Since $\mathbf{R}_1\mathbf{S}$ has rank M, this implies

$$\mathbf{V}\mathbf{A} = \mathbf{0} \qquad (23)$$

As $\mathbf{V}$ is $L \times P$ with rank L, there are $P - L = M$ independent *columns* which annihilate $\mathbf{V}$ from the right. But the M columns of the lower triangular matrix $\mathbf{A}$ not only annihilate $\mathbf{V}$, they are linearly independent as well. So *any annihilator* of $\mathbf{V}$ is in the column space of $\mathbf{A}$. In particular consider nonzero vectors of the form $\begin{pmatrix} \times \\ \mathbf{0} \end{pmatrix}$ where $\times$ has length $L+1$. The only vector of this form which annihilates $\mathbf{V}$ from the right is the 0th column of $\mathbf{A}$. This column (hence $c(n)$) can therefore be identified upto scale.

Does the method work when the data is contaminated by noise? In this case we have $\mathbf{y}(n) = \mathbf{A}\mathbf{R}_1\mathbf{s}(n) + \mathbf{e}(n)$, where $\mathbf{e}(n)$ is the blocked version of channel noise $e(n)$. Assume $\mathbf{s}(n)$ and $\mathbf{e}(n)$ are jointly wide-sense stationary and uncorrelated. Denoting the autocorrelation of $\mathbf{y}(n)$ by $\mathbf{R}_y$, and so forth, we then have

$$\mathbf{R}_y - \mathbf{R}_e = \mathbf{A}\mathbf{R}_1\mathbf{R}_s\mathbf{R}_1^\dagger\mathbf{A}^\dagger.$$

By repeating the preceding arguments about ranks and annihilators, it is readily verified that the matrix on the right has precisely L independent left-annihilators. Since the left hand side can be estimated from the received signal and noise statistics (assumed known), we can therefore estimate the L annihilators. These are also the annihilators of $\mathbf{A}$. So the channel can be identified as before.

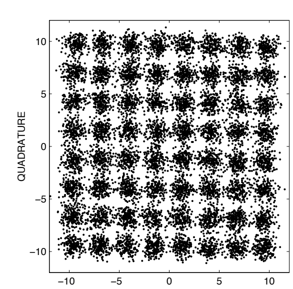

Figure 12. Result of equalization after blind identification.

Example. To demonstrate the idea with an example, we consider a simple 4th order FIR channel ($L = 4$) with $C(z)$ given by $-0.7684 - 0.8655z^{-1} + 0.4305z^{-2} - 0.3204z^{-3} + 0.4992z^{-4}$. We choose a transmitter using a 64-QAM constellation, and assume that $\mathbf{s}(n)$ is a blocked version with $M = 12$ so there are 12 subusers $s_k(n)$. Therefore $P = M + L = 16$. Assuming the noise $e(n)$ is white and the SNR at the channel output is 25 dB, we estimate the channel using the above method. Once the channel is estimated, it can be used in equalization. The scatter diagram for the equalized signal is shown in Fig. 12.

VII. CONCLUDING REMARKS

One problem which was not addressed in this review is the minimization of redundancy in the precoder.[4] First observe that in Fig. 1 if we are allowed to have infinite length equalizers then we can always use $1/C(e^{j\omega})$ to equalize the channel, as long as $C(e^{j\omega})$ does not have unit circle zeros. In this case there is no need for redundancy, in fact no need for the filter bank precoder at all. It is more interesting and desirable to have FIR equalizers and we found that the use of redundancy L (where L is the channel order) is sufficient. But is it possible to have smaller redundancy? The answer is indeed yes, under some conditions. The results are rather involved, and can be found in fairly recent papers [23], [13], [9], [12].

Another interesting aspect that was not addressed is resistance to channel nulls. To explain this idea recall that the inverse of the channel response $C(\rho_k)$ is often involved in the equalization, for example Eq. (12) and Eq. (16). If $C(\rho_k)$ is very small or zero then we have a problem. A simple way to overcome this difficulty is to add extra redundancy of L samples in the precoder (e.g., instead of adding L zeros in Fig. 6 add $2L$ zeros). In this way, the coefficients $C(\rho_k)$ are involved for $M+L$ different values of ρ_k. As shown in [7], this allows us a choice at the receiver to choose any M out of the $M + L$ values of $C(\rho_k)$ for equalization. Since $C(z)$ has order L, at most L of these $C(\rho_k)$'s could be zero and we can always pick the largest M coefficients $C(\rho_k)$ for equalization. Such designs are called null-resistant designs. Many details on this can be found in [4], [7].

Research on the topic of filter bank precoders is still growing. For example there are four papers on filter bank precoders in the Sept. 2003 issue of the *IEEE Transactions on Signal Processing*! In [19] we introduce a new *frequency domain method for blind identification*, which offers some practical advantages. There remain many interesting problems in this area as the reader will learn from the other papers in this special session.

[4]For large M the bandwidth expansion factor is small even for moderate L and this is not a serious issue.

VIII. REFERENCES

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