

MIMO radar, SIMO radar, and IFIR radar: a comparison

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Abstract—This paper focuses on a simple beamforming problem and compares the MIMO and SIMO radar systems for the case where the transmitter and receiver are collocated. The simplicity of the application allows one to see clearly where the advantages of MIMO radar come from, and what the tradeoffs are. The comparison also includes a third system called the IFIR radar (interpolated FIR radar). The MIMO radar offers advantages in terms of clutter reduction, especially when the extra coherent integration time can be exploited, but it does not offer an SNR advantage — in fact the SNR is worse than that of the SIMO radar when extra integration cannot be performed. The IFIR radar, which is essentially a simple variation of phased array radars, has an advantage over the SIMO radar in terms of clutter reduction, and does not compromise SNR in order to achieve this. The multiple orthogonal waveforms used in MIMO radar typically require extra bandwidth for fixed range resolution but the IFIR radar does not require extra bandwidth.¹

Index Terms—MIMO radar, beamforming, IFIR radar, clutter.

I. INTRODUCTION

MIMO radars [1]–[4] were introduced in 2003-04, and have been researched quite actively in recent years [5]–[15]. A number of these contributions address the question of when MIMO radars perform better than conventional (SIMO) radars [8]–[13]. This paper is in the same spirit but we focus on a simple beamforming problem and compare the two systems for the case where the transmitter and receiver are collocated. The simplicity of the application allows us to see clearly where the advantages come from, and what the tradeoffs are. In our comparison we also include a system called the IFIR radar, which is essentially a simple variation of the phased array radar with some clear advantages.

Sec. II briefly introduces the three systems and Sec. III formulates the beamforming issues. Sec. IV discusses the received clutter power for the three systems and Sec. V provides a calculation of signal and jammer powers for the three cases. Sec. VI describes the advantage obtained from the possibility of extra coherent integration that can be performed in MIMO radar (over and above what is possible in SIMO and IFIR radars). Sec. VII summarizes the detailed comparison of the three systems with respect to signal to noise ratio (SNR), signal to clutter ratio (SCR) and signal to jammer ratio (SJR) (Tables 1 and 2). We will see that in situations where extra

integration is not possible in MIMO radar, the IFIR method is the best among the three, whereas when extra integration is possible, the MIMO radar is the best in terms of reducing clutter (maximizing SCR). Next, as far as SNR is concerned MIMO radar, even with extra integration, offers no advantage over SIMO or IFIR radars, whereas with no extra integration its SNR is worse. On the other hand the MIMO radar does offer the possibility to reduce the effect of jammers (i.e., improve SJR), by taking advantage the waveform diversity that is inherent to MIMO radars.

II. THE MIMO, SIMO, AND IFIR RADARS

Fig. 1 (top) shows the collocated transmitting and receiving ULAs. The transmitter has N_t elements spaced apart by d_t and the receiver has N_r elements spaced apart by d_r . Throughout the paper we assume the receiver array elements have the standard spacing $d_r = \lambda/2$. For the SIMO radar $d_t = d_r = d$, and the N_t waveforms $s_k(t)$ are identical:

$$s_k(t) = s(t) \quad (1)$$

For the MIMO radar $d_t = N_r d_r$, and the waveforms are orthonormal:

$$\int s_k(t) s_m^*(t) dt = \delta[k - m] \quad (2)$$

Each of the N_r sensors at the receiver of the MIMO radar uses a bank of N_t matched filters [2], thereby resulting in $N_t N_r$ independent signals which define a virtual array of size $N_t N_r$, with effective interelement spacing d_r , as shown in the bottom of the figure. The third type of radar we consider is the IFIR radar which has $d_t = N_r d_r$ as in a MIMO radar, but the waveforms are identical ($s_k(t) = s(t)$) as in a SIMO radar. The term “IFIR” stands for “interpolated FIR filter”, and arises from analogy with a well known filter design technique [16] (Sec. III-C).

In the SIMO and IFIR cases, the transmitter and receiver both participate in beamforming, through the multipliers a_n and b_n . In the case of the MIMO radar, there is no beamforming at the transmitter, so all space is uniformly lit; beamforming is done at the receiver, using the $N_t N_r$ virtual array taps c_n shown in the figure (the multipliers b_n are not relevant in the MIMO radar case).

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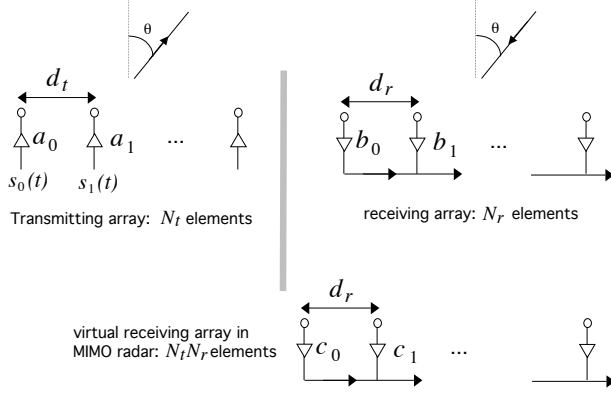


Fig. 1. Notations for transmitting and receiving arrays. The virtual receiving array for MIMO radar with $d_t = N_r d_r$ is shown in the bottom.

III. THE TRANSMITTING AND RECEIVING BEAMS

A. SIMO radar beams

With a_n and b_n representing the beamforming multipliers at the transmitter and receiver respectively, the transmitter and receiver beams are

$$H(e^{j\omega}) = \sum_{n=0}^{N_t-1} a_n e^{-j\omega n} \quad (3)$$

and

$$G(e^{j\omega}) = \sum_{n=0}^{N_r-1} b_n e^{-j\omega n} \quad (4)$$

where ω is the transformed angle variable

$$\omega = 2\pi d \sin \theta / \lambda, \quad (5)$$

θ is the angle with respect to vertical (Fig. 1), and λ is the wave length. We will consider the example where $a_n = 1$ for all n . Thus the beam is centered at $\theta = 0$. So

$$\sum_{n=0}^{N_t-1} a_n = N_t, \quad \text{and} \quad \sum_{n=0}^{N_t-1} |a_n|^2 = N_t \quad (6)$$

For the receiver we will normalize b_n such that

$$\sum_{n=0}^{N_r-1} |b_n|^2 = 1 \quad (7)$$

This provides unit gain for noise. Thus, in our example, $b_n = 1/\sqrt{N_r}$ so that the gain at $\theta = 0$ is

$$\sum_{n=0}^{N_r-1} b_n = \sqrt{N_r} \quad (8)$$

The beamwidths of the transmitter and receiver are approximately $2\pi/N_t$ and $2\pi/N_r$ (plotted against ω).

B. MIMO radar beams

For the MIMO radar also we choose $a_n = 1$ so that (6) still holds. But there is no beamforming at the transmitter. We now have $d_t = N_r d_r$ so that there is a virtual array of size $N_t N_r$. We choose the virtual array taps such that

$$\sum_{n=0}^{N_t N_r - 1} |c_n|^2 = 1 \quad (9)$$

so that the noise gain is normalized to unity. With $c_n = 1/\sqrt{N_t N_r}$ the beam is focused at $\theta = 0$ with gain

$$\sum_{n=0}^{N_t N_r - 1} c_n = \sqrt{N_t N_r} \quad (10)$$

The beamwidth is approximately $2\pi/N_t N_r$.

C. IFIR radar beams

For the IFIR case also, it is assumed (as in MIMO radar) that $d_t = N_r d_r$. But since $s_k(t) = s(t)$ for all k , the transmitter beam is

$$H(e^{j\omega N_r}) = \sum_{n=0}^{N_t-1} a_n e^{-j\omega N_r n} \quad (11)$$

With receiver beam as in (4), the transmit-receive beam is

$$H(e^{j\omega N_r}) G(e^{j\omega}) \quad (12)$$

If $H(e^{j\omega})$ represents a beam focused at $\theta = 0$, then $H(e^{j\omega N_r})$ has $N_r - 1$ extra lobes (grating lobes), separated in ω by $2\pi/N_r$. The receiver $G(e^{j\omega})$ tends to suppress the unwanted beams and keeps only the main beam at $\omega = 0$ (i.e., $\theta = 0$). This is similar to the use of an interpolated FIR (IFIR) digital filter to achieve narrow passband without unduly increasing the filter order. See [16] or Sec. 4.4.1 of [18]. The filter analogy is described in Fig. 2.

Since the main beam of $H(e^{j\omega N_r})$ is N_r times narrower than that of $H(e^{j\omega})$, the IFIR method allows us to achieve a much narrower main beam with only $N_t + N_r$ antenna elements and a single waveform $s(t)$. A beam of comparable narrowness can be achieved by a SIMO radar by using $N_t N_r$ antenna elements and one waveform, or by the MIMO radar by using $N_t + N_r$ elements and N_t waveforms.

We will again consider the example where $a_n = 1$ for all n . As in the SIMO case (6) holds, and furthermore b_n is normalized for unit energy as in (7). Thus $b_n = 1/\sqrt{N_r}$, and (8) holds. The main beamwidths of the transmitter and receiver are approximately $2\pi/N_t N_r$ and $2\pi/N_r$ (plotted against ω).

D. General remarks about the beams

Figure 3 gives a qualitative demonstration of how the beams work in the three cases. For the SIMO radar the element spacings are $d_t = d_r = \lambda/2$ and the transmitter and receiver beams are as in part (a) (if $N_t = N_r$). For the MIMO radar (part (b)) the virtual array helps to synthesize a very sharp receiver beam. For the IFIR radar the transmitter produces a very sharp main lobe, comparable to the MIMO radar, but there are extra grating lobes. These grating lobes are

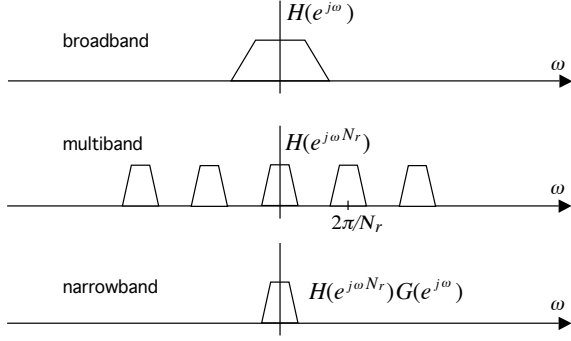


Fig. 2. Explanation of how the IFIR filter works. The filter $H(e^{j\omega})$ has one passband in $[-\pi, \pi]$, whereas $H(e^{j\omega N_r})$ has N_r passbands separated by $2\pi/N_r$. The filter $G(e^{j\omega})$ retains one of these N_r passbands. The result is a narrow band filter $H(e^{j\omega N_r})G(e^{j\omega})$, at a low cost proportional to that of the broadband filter $H(e^{j\omega})$.

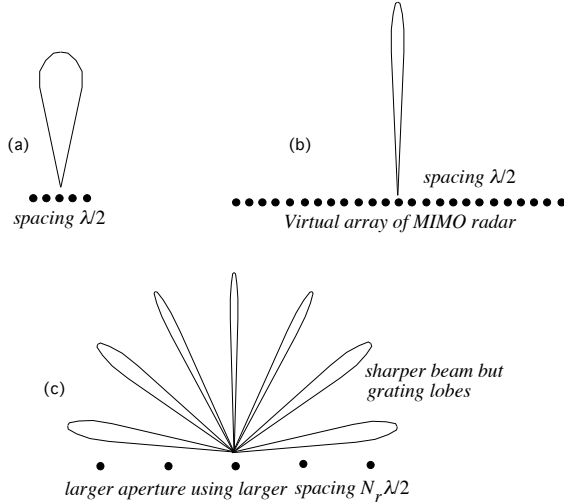


Fig. 3. (a) Element spacing and beams in SIMO radar, (b) virtual array and sharper beam in MIMO radar, and (c) transmitter array and beam in IFIR radar; the main lobe is sharp like a MIMO radar, and the grating lobes are removed by the receiver beam which is as in part (a).

suppressed by receiver beams which look like the one in part (a). Observe that the receiver can extract each one of the N_t transmitted lobes simultaneously by building N_t beamformers in parallel.

Notice that in all three systems the *transmitted power is identical* since the waveforms $s(t)$, and $s_k(t)$ are all assumed to have unit energy and furthermore $\sum_n |a_n|^2 = N_t$ in all cases. Similarly, the *received noise power* is identical in all three systems, because of the normalization conventions for $\{b_n\}$ and $\{c_n\}$.

In all comparisons we would like to assume a *point target*. More accurately, we will assume that the target is concentrated in a region of width $\Delta\phi$ (in the variable ω) where $\Delta\phi \ll 2\pi/N_t N_r$ with uniform amplitude (RCS) α . In our examples this will replace the point target assumption. The jammers will also be assumed to be “point jammers” in the above sense.

IV. CLUTTER POWER CALCULATION

A. Clutter in MIMO case

Let the clutter amplitude density at angle ω be $S_c(\omega)$. That is, $\int_{\omega_0-\epsilon}^{\omega_0+\epsilon} S_c(\omega) d\omega/2\pi$ is the clutter amplitude in the angular range $[\omega_0 - \epsilon, \omega_0 + \epsilon]$. Physically, this means that the clutter reflects the field $S_c(\omega) a_k s_k(t)$ in response to the signal from the k th transmitting antenna. Only the k th matched filter sees this (because of (2) and because $a_k = 1$), and its output is $S_c(\omega)$ from “angle” ω . So the virtual array output contributed from clutter at angle ω is $S_c(\omega) C(e^{j\omega})$ where $C(e^{j\omega}) = \sum_n c_n e^{-j\omega n}$. The total contribution from all angles is

$$y_c = \int S_c(\omega) C(e^{j\omega}) \frac{d\omega}{2\pi} \quad (13)$$

This is a random variable since $S_c(\omega)$ is a random number for each ω . We now assume that the clutter correlation function $E[S_c(\omega) S_c^*(\omega')]$ has the form

$$E[S_c(\omega) S_c^*(\omega')] = R_{s_c}(\omega - \omega') \quad (14)$$

That is, it depends only on the difference $\omega - \omega'$; this can be regarded as “wide sense stationarity” in the variable ω . Then

$$\begin{aligned} E|y_c|^2 &= \frac{1}{(2\pi)^2} \int \int R_{s_c}(\omega - \omega') C(e^{j\omega}) C^*(e^{j\omega'}) d\omega d\omega' \\ &= \frac{1}{(2\pi)^2} \int R_{s_c}(v) dv \int C(e^{j\omega}) C^*(e^{j(\omega-v)}) d\omega \\ &= \frac{1}{2\pi} \int R_{s_c}(v) R_c(e^{jv}) dv \end{aligned}$$

So the autocorrelation

$$R_c(e^{jv}) = \int C(e^{j\omega}) C^*(e^{j(\omega-v)}) \frac{d\omega}{2\pi}$$

of $C(e^{j\omega})$ is the determining factor. For the special case where the clutter is uncorrelated from point to point in space, we have

$$R_{s_c}(v) = 2\pi\sigma_c^2 \delta(v), \quad (15)$$

so that $E|y_c|^2 = \sigma_c^2 R_c(e^{j0}) = \sigma_c^2 \int |C(e^{j\omega})|^2 d\omega/2\pi = \sigma_c^2 \sum c(n)|^2$. Using $\sum c(n)|^2 = 1$ we therefore have

$$E|y_c|^2 = \sigma_c^2 \quad (\text{MIMO clutter}) \quad (16)$$

B. Clutter in SIMO case

Next consider the SIMO radar: the clutter $S_c(\omega)$ reflects $S_c(\omega) H(e^{j\omega})$, which is received at each receiving sensor from angle ω . Thus the output of receiving beamformer is $S_c(\omega) H(e^{j\omega}) G(e^{j\omega})$ at angle ω . So the total output clutter variance is $E|y_c|^2 = \int R_{s_c}(v) R_d(e^{jv}) dv/2\pi$ where $R_d(e^{j\omega})$ is the autocorrelation of $H(e^{j\omega}) G(e^{j\omega})$. For the special case of uncorrelated clutter this reduces to

$$E|y_c|^2 = \sigma_c^2 \int |H(e^{j\omega}) G(e^{j\omega})|^2 \frac{d\omega}{2\pi} = \sigma_c^2 \sum_n |d_n|^2 \quad (17)$$

where $d_n = (a_n * b_n)$. For the special case where $N_t = N_r$, $a_n = 1$, and $b_n = 1/\sqrt{N_r}$ as before, the convolution d_n is a symmetric triangle with energy

$$\sum_n |d_n|^2 = N_r + \frac{(N_r - 1)(2N_r - 1)}{3} = \frac{2N_r^2 + 1}{3} \approx \frac{2N_r^2}{3}$$

So the clutter power becomes approximately

$$E|y_c|^2 = \frac{2\sigma_c^2 N_r^2}{3} \quad (\text{SIMO clutter}) \quad (18)$$

The received clutter power in the SIMO case grows quadratically with N_r unlike in MIMO radar where the clutter power (16) is not amplified at all!

C. Clutter in IFIR case

Assuming the clutter to be as in (15), the clutter power received is $\sigma_c^2 \sum_n |d_n|^2$, where d_n has transform

$$D(e^{j\omega}) = H(e^{j\omega N_r})G(e^{j\omega}) \quad (19)$$

Note that the sequence d_n has $N_t N_r$ samples given by

$$a_n b_m, \quad 0 \leq n \leq N_t - 1, 0 \leq m \leq N_r - 1$$

So

$$\sum_{n=0}^{N_t N_r - 1} |d_n|^2 = \sum_{n=0}^{N_t - 1} |a_n|^2 \sum_{m=0}^{N_r - 1} |b_m|^2 = N_t$$

so that the received clutter power is

$$E|y_c|^2 = \sigma_c^2 N_t \quad (\text{IFIR clutter}) \quad (20)$$

This is intermediate between the SIMO and MIMO cases.

D. Remark on correlated clutter

Now consider an extreme hypothetical case where

$$E[S_c(\omega)S_c^*(\omega')] = K \quad (21)$$

that is, the clutter is completely correlated between any two angular points. Taking the MIMO radar as an example,

$$\begin{aligned} E|y_c|^2 &= K \int C(e^{j\omega}) \frac{d\omega}{2\pi} \int C^*(e^{jv}) \frac{dv}{2\pi} \\ &= K |c_0|^2 = \frac{K}{N_t N_r} \end{aligned} \quad (22)$$

That is, the output clutter power depends only on c_0 ! To add insight to this behavior, consider deterministic clutter, with $S_c(\omega) = \sqrt{K}$ for all ω . Then $y_c = \sqrt{K} \int C(e^{j\omega}) d\omega / 2\pi = c_0 \sqrt{K}$, so $|y_c|^2 = K |c_0|^2$ which is a similar conclusion. The integrated effect of $S_c(\omega)$ cancels at all sensors except the 0th sensor corresponding to c_0 . For example consider $c_n, n \neq 0$. The fields $e^{-j\omega n}$ integrated over all ω simply cancel because $\int e^{-j\omega n} d\omega = 0$ for $n \neq 0$. What is special about c_0 is that all points in the range bin are equally distant from the 0th sensor. So they add in phase. In practice there is leakage from neighboring range bins, so this will not happen so exactly. Strictly speaking, one should regard $S_c(\omega)$ as the total clutter amplitude from angle ω , taking leakage from neighboring range bins into account.

V. SIGNAL AND NUISANCE POWERS AT RECEIVER

We now proceed to calculate the received signal, noise, and jammer powers.

A. Signal and nuisance powers, SIMO case

With the beams focused on a target at $\theta = 0^\circ$, the amplitude of signal received from the target is approximately

$$\alpha \Delta \phi \sum_{n=0}^{N_t-1} a_n \sum_{n=0}^{N_r-1} b_n = \alpha \Delta \phi N_t \sqrt{N_r} \quad (23)$$

So the received signal power from target is

$$|\alpha \Delta \phi|^2 N_t^2 N_r \quad (24)$$

With uniform noise variance σ_e^2 , the received noise power is $\sigma_e^2 \sum_n |b_n|^2 = \sigma_e^2$. Next consider jammer power. Let α_J be the amplitude of jammer in an angular region of width $\Delta \phi$. Then the received jammer power is

$$\left| \alpha_J \Delta \phi \sum_n b_n e^{-j\omega_J n} \right|^2 \quad (25)$$

In our examples we will set $\omega_J = 0$ (worst position for jammer) so the received jammer power is²

$$\left| \alpha_J \Delta \phi \sum_n b_n \right|^2 = \left| \alpha_J \Delta \phi \right|^2 N_r \quad (26)$$

where we have substituted the value $b_n = 1/\sqrt{N_r}$ given earlier. Summarizing, with $a_n = 1$ and $b_n = 1/\sqrt{N_r}$, the received signal has the following power components:

- 1) Signal power from target = $|\alpha \Delta \phi|^2 N_t^2 N_r$
- 2) Clutter power = $2\sigma_c^2 N_t^2 / 3$ (for $N_t = N_r$; see Eq. (18)).
- 3) Noise power = σ_e^2 .
- 4) Jammer power = $|\alpha_J \Delta \phi|^2 N_r$.

B. Signal and nuisance powers, MIMO case

The k th transmitting antenna sends $a_k s_k(t)$. Think of a_k as an allpass filter in ω (where ω is as in (5)) and target as a narrow band signal confined to bandwidth $\Delta \phi$. So the response from target is $a_k \times \alpha \times \Delta \phi s_k(t)$. This produces a response only at the output of the k th receiving matched filter, and this output is $a_k \alpha \Delta \phi = \alpha \Delta \phi$ with $a_k = 1$. The output of the virtual array is therefore

$$\alpha \Delta \phi \sum_{k=0}^{N_t N_r - 1} c_k = \alpha \Delta \phi \sqrt{N_t N_r} \quad (27)$$

because the target is concentrated near $\omega = 0$, and the virtual beam is focused in that direction. Thus the signal power is

$$|\alpha \Delta \phi|^2 N_t N_r \quad (28)$$

This is smaller than the signal power (24) in the SIMO case. Next, the noise power is simply $\sigma_e^2 \sum_n |c_n|^2 = \sigma_e^2$, as in the SIMO case. So the SNR in MIMO radar is N_t time smaller than that in the SIMO radar.

With β_n denoting the output of the matched filter $s_n^*(-t)$ in response to the jammer waveform $s_J(t)$ (Fig. 4), the received jammer power has the form

$$\left| \alpha_J \Delta \phi \sum_n c_n \beta_n e^{-j\omega_J n} \right|^2 \quad (29)$$

²If the jammer is in a direction different from the desired signal, one inserts nulls in those directions. In this paper we do not discuss this case.

Note that

$$\beta_n = \int s_J(t) s_n^*(t) dt \quad (30)$$

where it is assumed for simplicity that the matched filters sample the output at $t = 0$. So the *jammer sees a modified version of the virtual array taps* $\{c_n\}$. In general the jammer signal has the form

$$s_J(t) = \sum_{k=0}^{N_t-1} \beta_k s_k(t) + \beta' s_{\perp}(t) \quad (31)$$

where $s_{\perp}(t)$ is a unit energy signal in the orthogonal complement of the space spanned by the waveforms $\{s_k(t)\}$.

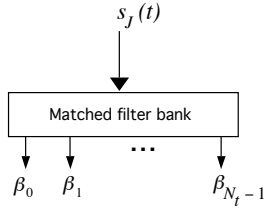


Fig. 4. Matched filter outputs β_k in response to the jammer signal $s_J(t)$.

We assume without loss of generality that $s_k(t)$ and $s_J(t)$ all have unit energy (the free parameter α_J can be used to describe the strength of the jammer). For convenience we extend the definition of β_k periodically with period N_t in the variable k , so that the equivalent tap values of the virtual array as seen by the jammer are $c_n \beta_n$, with $\beta_{n+N_t} = \beta_n$. Summarizing, with $a_n = 1$ and $c_n = 1/\sqrt{N_r N_t}$, the received signal has the following power components in the MIMO radar case:

- 1) Signal power from target = $|\alpha \Delta \phi|^2 N_t N_r = |\alpha \Delta \phi|^2 N_t^2$ for $N_t = N_r$.
- 2) Clutter power = σ_e^2 (from Eq. (16)).
- 3) Noise power = σ_e^2 .
- 4) Jammer power = $|\alpha_J \Delta \phi \sum_n c_n \beta_n|^2$ (assuming $\omega_J = 0$). This becomes $|\alpha_J \Delta \phi|^2 N_t$ for the case where $N_t = N_r$ and β_k takes worst case values (see below).

Worst case β_k . Since $\{s_k(t)\}$ is orthonormal it follows from (31) that

$$\sum_{k=0}^{N_t-1} |\beta_k|^2 + |\beta'|^2 = 1 \quad (32)$$

Using Cauchy-Schwartz inequality we find

$$\begin{aligned} \left| \sum_{n=0}^{N_t-1} \beta_n \right|^2 &\leq \sum_{n=0}^{N_t-1} |\beta_n|^2 \sum_{n=0}^{N_t-1} 1 \\ &\leq \left(\sum_{n=0}^{N_t-1} |\beta_n|^2 + |\beta'|^2 \right) N_t = N_t \end{aligned} \quad (33)$$

Thus

$$\left| \sum_{n=0}^{N_t-1} \beta_n \right|^2 \leq N_t. \quad (34)$$

Equality corresponds to worst-case jamming.

C. Signal and nuisance powers, IFIR case

The amplitude of signal received from the target is approximately

$$\alpha \Delta \phi \sum_{n=0}^{N_t-1} a_n \sum_{n=0}^{N_r-1} b_n = \alpha \Delta \phi N_t \sqrt{N_r} \quad (35)$$

So the received signal power from target is

$$|\alpha \Delta \phi|^2 N_t^2 N_r \quad (36)$$

With uniform noise variance σ_e^2 , the received noise power is $\sigma_e^2 \sum_n |b_n|^2 = \sigma_e^2$. Let α_J be the amplitude of jammer, which is concentrated in an angular region of width $\Delta \phi$. Then the received jammer power is

$$\left| \alpha_J \Delta \phi \sum_n b_n \right|^2 = \left| \alpha_J \Delta \phi \right|^2 N_r \quad (37)$$

where we have used $\omega_J = 0$, and substituted the value $b_n = 1/\sqrt{N_r}$ given earlier. Summarizing, with $a_n = 1$ and $b_n = 1/\sqrt{N_r}$, the received signal has the following components for IFIR radar:

- 1) Signal power from target = $|\alpha \Delta \phi|^2 N_t^2 N_r$
- 2) Clutter power = $\sigma_e^2 N_t$ (from (20)).
- 3) Noise power = σ_e^2 .
- 4) Jammer power = $|\alpha_J \Delta \phi|^2 N_r$.

Remark on SIMO and IFIR jammers. In the SIMO case also we have to introduce a factor β because the jammer signal $s_J(t)$ is filtered by the matched filter $s^*(-t)$:

$$\beta = \int s_J(t) s^*(t) dt$$

So the jammer power should actually be $|\beta \alpha_J \Delta \phi|^2 N_r$. Similarly in the IFIR case also, there is a factor of $|\beta|^2$ in the jammer power. With the energies of $s_J(t)$ and $s(t)$ normalized to unity, $|\beta|^2 < 1$. Since β is in general unknown, we will set $\beta = 1$ (worst case) in most discussions.

VI. INTEGRATION TIME ON TARGET

In pulsed radars, the signal from a target is integrated over several pulses to increase the signal to nuisance ratios (see Sec. 2.6 in [17]). Now, in MIMO radars the transmitter illuminates all space but in SIMO, only about $(1/N_t)$ th of the space (in terms of the variable ω) is illuminated. So in MIMO, we can in theory stay on target for a longer time by some factor P . This means, for a given range bin, we can sum over P times as many pulses as we normally do. P will be referred to as the *extra coherent integration factor*. Ideally $P = N_t$ but in practice it is possible that $P < N_t$ or that extra integration is not possible ($P = 1$). Fig. 5 shows an example where the SIMO radar integrates over four pulses and the MIMO radar over 12 pulses. In this example $P = 12/4 = 3$. Notice that this extra integration (or extra summation over more pulses) does not affect any of the standard parameters such as the peak power, average power, or the duty cycle τ/T_p .

To understand how P affects SNR and other ratios, consider a simple example where we have a received signal

$$s_k(t) + s_J(t) + s_c(t) + e(t) \quad (38)$$

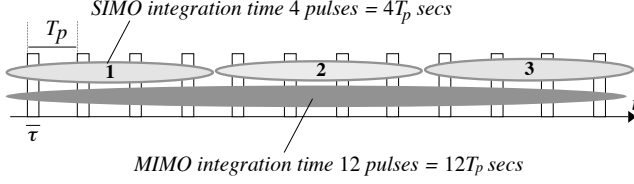


Fig. 5. The radar pulse waveform, and the extra integration potentiality in MIMO radar. See text.

Here $s_k(t)$ is the transmitted signal, $s_J(t)$ the jammer, $s_c(t)$ the clutter, and $e(t)$ the noise. At the output of the matched filter $s_k^*(-t)$, the components after sampling at the ideal instant are as follows:

- 1) Signal amplitude: $\int |s_k(t)|^2 dt$.
- 2) Jammer amplitude: $\int s_J(t) s_k^*(t) dt$.
- 3) Clutter amplitude: $\int s_c(t) s_k^*(t) dt$. So the clutter variance is $\sigma_c^2 \int |s_k(t)|^2 dt$.
- 4) Noise amplitude: $\int e(t) s_k^*(t) dt$. So the noise variance is $\sigma_e^2 \int |s_k(t)|^2 dt$.

If we can coherently integrate over P pulses the signal amplitudes add whereas the noise and clutter powers add:

- 1) Signal amplitude: $P \int |s_k(t)|^2 dt$, assuming negligible loss due to incoherence [17].
- 2) Jammer amplitude: $P \int s_J(t) s_k^*(t) dt$ if the jammer components seen at different pulse times are fully correlated; otherwise the jammer power increases by the factor P .
- 3) Clutter variance: $P \sigma_c^2 \int |s_k(t)|^2 dt$. This assumes that the clutter components seen at different pulse times are uncorrelated.
- 4) Noise variance: $P \sigma_e^2 \int |s_k(t)|^2 dt$.

In short, after integration the signal and jammer powers increase by a factor of P^2 whereas clutter and noise powers increase only by P . This statement is true at the output of every virtual sensor, that is, at the input of each tap c_n . So the beamformer output which is a linear combination of the tap inputs will also see the exact same situation. Using the above example as a guideline we see that the various powers become as follows for the MIMO case:

- 1) Signal power from target = $P^2 |\alpha \Delta \phi|^2 N_t N_r$
- 2) Clutter power = $P \sigma_c^2$.
- 3) Noise power = $P \sigma_e^2$.
- 4) Jammer power with $\omega_J = 0$, and using $c_n = 1/\sqrt{N_t N_r}$

$$\begin{aligned}
 &= P^2 |\alpha_J \Delta \phi|^2 \sum_n c_n \beta_n^2 \\
 &= (P^2 / N_t N_r) \left| \alpha_J \Delta \phi \sum_{n=0}^{N_t N_r - 1} \beta_n \right|^2 \\
 &= (P^2 N_r / N_t) \left| \alpha_J \Delta \phi \sum_{n=0}^{N_t - 1} \beta_n \right|^2 \quad (39)
 \end{aligned}$$

In the IFIR case also, the main transmitter beam has width N_r times smaller than the conventional SIMO beam. In the scanning mode the space to be scanned by this beam is N_r

times smaller than the total angular region because there are N_r copies. So, the time on target is the same as in the SIMO case. No extra integration is possible for IFIR radar.

VII. COMPARISONS

We are now ready to summarize the comparison of SIMO, MIMO, and IFIR radars. To be specific we shall assume that $N_t = N_r$. When MIMO radar allows extra integration we assume it is full, coherent, integration ($P = N_t$). When no extra integration is allowed, $P = 1$. For simplicity we refer to these two cases as “with integration” and “without integration” respectively. We also assume that β_k in the MIMO radar assumes the worst case value so that (34) holds with equality. For the SIMO radar we have from Sec. V:

- 1) Signal power from target = $|\alpha \Delta \phi|^2 N_t^3$
- 2) Clutter power = $2 \sigma_c^2 N_t^2 / 3$.
- 3) Noise power = σ_e^2 .
- 4) Jammer power = $|\alpha_J \Delta \phi|^2 N_t$,

and for the MIMO radar with integration:

- 1) Signal power from target = $|\alpha \Delta \phi|^2 N_t^4$
- 2) Clutter power = $N_t \sigma_c^2$ (if clutter is uncorrelated between different pulses) or $N_t^2 \sigma_c^2$ (if clutter is totally correlated between different pulses).
- 3) Noise power = $N_t \sigma_e^2$.
- 4) Jammer power = $|\alpha_J \Delta \phi|^2 N_t^3$ (for deterministic α_J in jammer), or $|\alpha_J \Delta \phi|^2 N_t^2$ (for random jammer).

Finally for the IFIR radar with $N_t = N_r$, we have:

- 1) Signal power from target = $|\alpha \Delta \phi|^2 N_t^3$
- 2) Clutter power = $\sigma_c^2 N_t$
- 3) Noise power = σ_e^2 .
- 4) Jammer power = $|\alpha_J \Delta \phi|^2 N_t$.

Pertaining to the “random jammer” addition in the MIMO case, note that the jammer amplitude as seen by the matched filter can be random or uncorrelated from pulse to pulse. If the jammer has random α_J with uncorrelated values from pulse to pulse, then in the extra integration phase the jammer power increases by P instead of P^2 . (In this case $|\alpha_J|^2$ is to be interpreted as the mean square value $E[|\alpha_J|^2]$ of the jammer amplitude.) Table 1 shows the signal to clutter ratio (SCR), signal to noise ratio (SNR), signal to jammer ratio (SJR) for the SIMO radar, the MIMO radar with and without the P -fold extra integration, and the IFIR radar. Table 2 shows the improvement ratio over SIMO, of three methods, namely MIMO with and without extra integration, and IFIR. We now discuss the main points conveyed by the tables.

A. SIMO versus MIMO radar

When we compare the entries for MIMO and SIMO radars, the following points emerge:

- 1) MIMO radar (with or without extra integration) has better signal to clutter ratio than SIMO radar. This improvement is proportional to N_t when $P = 1$ (no extra integration), and to N_t^2 when $P = N_t$ (full coherent extra integration).
- 2) MIMO radar has N_t times smaller SNR than SIMO radar when there is no extra integration. With extra integration, MIMO radar has the same SNR as SIMO.

TABLE 1 Signal to nuisance ratios

	MIMO with integration	IFIR	SIMO	MIMO with no integration
SCR	$\frac{ \alpha\Delta\phi ^2 N_t^3}{\sigma_c^2}$ $\frac{ \alpha\Delta\phi ^2 N_t^2}{\sigma_c^2}$ (<i>t</i> -correl clutter)	$\frac{ \alpha\Delta\phi ^2 N_t^2}{\sigma_c^2}$	$\frac{3 \alpha\Delta\phi ^2 N_t}{2\sigma_c^2}$	$\frac{ \alpha\Delta\phi ^2 N_t^2}{\sigma_c^2}$
SNR	$\frac{ \alpha\Delta\phi ^2 N_t^3}{\sigma_e^2}$	$\frac{ \alpha\Delta\phi ^2 N_t^3}{\sigma_e^2}$	$\frac{ \alpha\Delta\phi ^2 N_t^3}{\sigma_e^2}$	$\frac{ \alpha\Delta\phi ^2 N_t^2}{\sigma_e^2}$
SJR	$ \frac{\alpha}{\alpha_J} ^2 N_t^2$ (random uncorrel. jammer) $ \frac{\alpha}{\alpha_J} ^2 N_t$ (deterministic jammer)	$ \frac{\alpha}{\alpha_J} ^2 N_t^2$	$ \frac{\alpha}{\alpha_J} ^2 N_t^2$	$ \frac{\alpha}{\alpha_J} ^2 N_t$

TABLE 2 MIMO and IFIR improvement over SIMO

	MIMO with integration	IFIR	MIMO with no integration
SCR	$2N_t^2/3$ $2N_t/3$ (<i>t</i> -correl clutter)	$2N_t/3$	$2N_t/3$
SNR	1	1	$1/N_t$
SJR	1 (random uncorrel. jammer) $1/N_t$ (deterministic jammer)	1	$1/N_t$

- 3) As long as there is extra integration, the MIMO radar is better in the case of random jammer amplitude α_J . Even for deterministic jammer this might be so because the worst case $\{\beta_k\}$ (which is what is considered in the table) happens rarely. Furthermore, clever manipulation of β_k can reduce the effect of jammer considerably. This can be achieved if the transmitted set of orthonormal waveforms $\{s_k(t)\}$ is frequently varied in a random fashion.
- 4) Regardless of whether we do the extra integration or not, MIMO radar always has the advantage of a sharper beam because of the longer virtual array; this also means increased freedom ($N_t N_r$ values for taps c_n). The IFIR radar also offers a sharper transmit-receive beam but the number of freedoms is not $N_t N_r$, but only $N_t + N_r$.

Practical difficulties with extra integration. The preceding discussions clearly show that MIMO radars have worse SNR compared to SIMO radars unless we are able to take advantage of the extra integration time. In practice, this may not always be possible. For example, if we are interested only in a

limited range of angles then the extra integration factor P is correspondingly smaller. In target tracking applications the extra integration advantage may disappear altogether. Furthermore, extra integration offers an advantage only under the assumptions that the target and the medium are coherent (do not fluctuate) during this time. Such assumptions can be unrealistic in practice. For further discussions the reader should refer to the paper by Daum and Huang [10].

The above difficulties notwithstanding, the preceding discussions show that MIMO radars do have an advantage over SIMO radars from the viewpoint of signal to clutter ratio, even if we do not perform the extra integration.

B. IFIR radar versus others

Here is a summary of how the IFIR radar compares with others.

- 1) IFIR is always better than SIMO (because of better SCR).
- 2) IFIR is better than MIMO without extra integration (because of better SNR and SJR).

- 3) IFIR is as good as MIMO with extra integration, as far as SNR and SJR go.
- 4) For SCR, IFIR is in between SIMO and MIMO with extra integration. For the case where clutter has high correlation from pulse to pulse, IFIR radar is as good as MIMO radar with integration, in all three respects! IFIR is even better than MIMO radar with integration, if jammer has correlation from pulse to pulse.

In situations where MIMO radars cannot be implemented with extra integration, IFIR is strictly better than MIMO. In fact IFIR is the best choice in that case. It has the same complexity as SIMO, and the best performance among MIMO, SIMO, and IFIR.

C. Doppler processing

In this paper we have not considered Doppler processing which is a major technique to reduce clutter. So we conclude this section with some remarks on Doppler. Referring to Fig. 5 which describes the radar pulse train, recall that the MIMO radar can typically integrate (or sum) over a larger number of pulses. This can be represented by the equation $y(n) = \sum_{k=0}^{N-1} f(k)x(n-k)$ where the value of N is P times larger for the MIMO radar. Now, this summation over several pulses can be regarded as Doppler filtering.³ Since the taps $f(k) = 1$ for all k , we have a lowpass Doppler filter which is appropriate for the special case of stationary clutter (for which Doppler frequency = 0). So we see that the improvement in signal to clutter ratio offered by the MIMO radar (Table 1) can be regarded as a direct consequence of the extra number of taps available for Doppler filtering. If the Doppler frequency is different from zero, we would of course choose the taps $f(k)$ differently, but in any case, it is clear that the extra integration time available for MIMO radar essentially implies that the Doppler filters can have more taps, and hence filter out clutter better (i.e., the Doppler filters can have narrower passbands and/or better stop band attenuation).

VIII. CONCLUDING REMARKS

An important issue not mentioned above is the question of bandwidth. The MIMO radar uses N_t waveforms $s_k(t)$, typically orthogonal. If orthogonality is achieved by staggering the waveforms in frequency domain (as in FDMA) then for a given range resolution, the required bandwidth increases. If on the other hand we use the same total bandwidth as in SIMO radar, then each MIMO waveform $s_k(t)$ has a smaller bandwidth individually which reduces the range resolution. It is possible to take a TDMA approach where the waveforms $s_k(t)$ are staggered in time. Since the total time taken up is then N_t times that of the SIMO radar, the MIMO radar again compromises range resolution (for fixed bandwidth). So the familiar tradeoff between bandwidth and range resolution should be considered carefully when comparing these systems.

³This is the case where Doppler filtering is decoupled from spatial filtering. But when the doppler-angle plane is nonseparable as in moving-platform radars, removing clutter is less trivial, and the use of STAP is more appropriate.

A discussion of FDMA, TDMA, and CDMA approaches to MIMO radar waveform design can be found in [11]. Finally it should be noticed that the IFIR radar, which is essentially a phased array radar with a slightly different transmitter array geometry, offers advantages in terms of sharpness of the beam and clutter reduction.

Another interesting point for future consideration is the joint optimization of the transmitter beamforming coefficients $\{a_n\}$ and receiver beamforming coefficients $\{b_n\}$ in the case of the SIMO and IFIR beamformers, based on our knowledge of the clutter map $S_c(\omega)$. An objective function such as the received signal to clutter power can be maximized by such optimization. Similarly the coefficients $\{c_n\}$ in the virtual array of the MIMO radar can be optimized for the same objective. A comparison of such optimized SIMO, MIMO, and IFIR radars for fixed transmitter power and fixed receiver noise gain can be very interesting.

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REFERENCES

- [1] D. W. Bliss and K. W. Forsythe, "Multiple-input multiple-output radar and imaging: degrees of freedom and resolution," 37th Asilomar Conf. on Sig., Sys., and Comp., pp. 54-59, Nov. 2003.
- [2] K. W. Forsythe, D. W. Bliss, and G. S. Fawcett, "MIMO Radar: performance issues," 38th Asilomar Conf. on Sig., Sys., and Comp., pp. 310-315, Nov. 2004.
- [3] F. C. Robey, S. Coutts, D. Weikle, J. C. McHarg, and K. Cuomo, "MIMO Radar Theory and Experimental Results," 38th Asilomar Conf. on Sig., Sys., and Comp., pp. 300-304, Nov. 2004.
- [4] E. Fishler, A. Haimovich, R. S. Blum, L. J. Cimini, D. Chizhik, and R. A. Valenzuela, "Performance of MIMO radar systems: Advantages of angular diversity," in Proc. IEEE Asilomar Conf. Sig., Sys., and Comp., pp. 305-309, Nov. 2004.
- [5] G. San Antonio and D. R. Fuhrmann, "Beampattern synthesis for wide-band MIMO radar systems," 21st IEEE Intl. Workshop on Comp. Adv. in multisensor adap. proc., Dec. 2005.
- [6] J. Li, and P. Stoica, "MIMO radar with colocated antennas", IEEE Sig. Proc. Mag., pp. 106-114, Sept. 2007.
- [7] A. M. Haimovich, R. S. Blum, and L. J. Cimini, Jr., "MIMO Radar with widely separated antennas", IEEE Sig. Proc. Mag., pp. 116-129, Jan. 2008.
- [8] Chun-Yang Chen and P. P. Vaidyanathan, "MIMO radar space-time adaptive processing using prolate spheroidal wave functions", IEEE Trans. Signal Processing, pp. 623-635, Feb., 2008.
- [9] Y. I. Abramovich and G. J. Frazer, "Bounds on the volume and height distributions for the MIMO radar ambiguity function", IEEE Signal Processing Letters, pp. 505-508, 2008.
- [10] F. Daum and J. Huang, "MIMO radar: snake oil or good idea?", Proc. IEEE Asil. Conf. on Sig., Sys., and Comp., Monterey, CA, Nov. 2008.
- [11] D. J. Rabideau, "Adaptive MIMO radar waveforms," IEEE Radar Conference, May 2008.
- [12] D. W. Bliss, K. W. Forsythe, S. K. Davis, G. S. Fawcett, D. J. Rabideau, L. L. Horowitz, and S. Kraut, "GMTI MIMO radar," Intl. WD&D Conf., pp. 118-122, 2009.
- [13] B. Friedlander, "On the relationship between MIMO and SIMO radars", IEEE Trans. Signal Processing, pp. 394-398, Jan., 2009.
- [14] A. Hassanien and S. A. Vorobyov, "Transmit/receive beamforming for MIMO radar with colocated antennas", Proc. IEEE Intl. Conf. Acoust., Speech, and Signal Proc., Taipei, 2009.
- [15] J. Li and P. Stoica (editors), *MIMO radar signal processing*, John Wiley & Sons, 2009.
- [16] Y. Neuvo, C.-Y. Dong, and S. K. Mitra, "Interpolated finite impulse response filters", IEEE Trans. ASSP, pp. 563-570, June 1984.
- [17] M. I. Skolnik, *Introduction to radar systems*, Mc-Graw Hill, 2001.
- [18] P. P. Vaidyanathan, *Multirate systems and filter banks*, Englewood Cliffs, NJ: Prentice-Hall, 1993.