A NEW EIGENFILTER BASED METHOD FOR OPTIMAL DESIGN OF CHANNEL SHORTENING EQUALIZERS

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ABSTRACT

Recently much attention has been given to the design of time-domain equalizers or TEQs for discrete multitone modulation (DMT) systems. In this paper, we present a new method for the design of such equalizers which minimizes both the intersymbol interference (ISI) and noise power observed in a DMT system. Furthermore, we show how this method can be used for the design of fractionally spaced equalizers or FSEs. Experimental results are presented showing that our design method performs better than other known techniques in terms of achievable bit rate. [1]

1. INTRODUCTION

In recent years, one problem which has been of great interest has been the design of time-domain equalizers or TEQs for discrete multitone modulation or DMT systems [1, 4, 6]. Due to the long impulse response of typical channels encountered in DMT systems such as twisted pair telephone lines [7], TEQs are needed to *shorten* the overall channel response to one sample more than the length of the cyclic prefix used.

Several methods previously proposed for the design of such TEQs involve the design of the effective channel and not the equalizer coefficients directly [1, 4]. With these methods, the equalizer coefficients must then be chosen to best fit the desired optimal effective channel. A new method, however, was recently introduced [6] which deals directly with the equalizer coefficients. In this method, the objective was to minimize the delay spread of the overall channel. The optimal equalizer coefficients were found to be related to an eigenvector of a particular matrix. In [8], we generlized this method to also account for the noise present in the system. The equalizer coefficients were chosen to minimize an objective function consisting of a convex combination of a channel shortening objective and a noise-to-signal ratio objective.

In this paper, we consider minimizing a different objective function consisting of a weighted sum of the intersymbol interference (ISI) power and the output noise power. Much like the methods of [6, 8], the optimal equalizer coefficients will also be found to be related to an eigenvector of a particular matrix. Furthermore, we will show that our results can be extended for the design of fractionally spaced equalizers or FSEs. Though FSEs have not been traditionally used as TEQs for DMT systems, the results shown here give merit to their possible future use.

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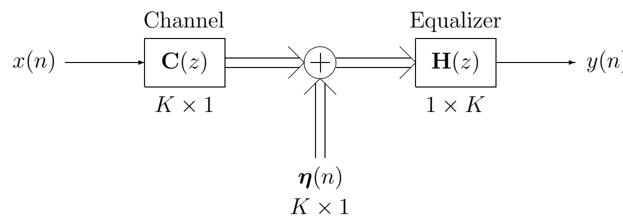

Fig. 1. SIMO-MISO channel and equalizer model.

2. THE TEQ DESIGN PROBLEM

Suppose we have the single-input multiple-output (SIMO) channel and multiple-input single-output (MISO) equalizer model shown in Figure 1. When $K = 1$ we obtain the classical symbol spaced equalizer (SSE) model. In Section 4 we show that the fractionally spaced equalizer (FSE) is a special case of this model. We make the following assumptions here.

- The channel $\mathbf{C}(z)$ is FIR of length L_c.
- The equalizer $\mathbf{H}(z)$ is FIR of length L_e.
- The input $x(n)$ is zero-mean and white with variance σ_x^2.
- The noise vector sequence $\boldsymbol{\eta}(n)$ is a zero-mean WSS random process uncorrelated with $x(n)$ and with autocorrelation sequence $\mathbf{R}_{\boldsymbol{\eta}\boldsymbol{\eta}}(k)$.

Denote the impulse responses of $\mathbf{C}(z)$ and $\mathbf{H}(z)$ by $\mathbf{c}(n)$ and $\mathbf{h}(n)$, respectively. The effective channel is $c_{\text{eff}}(n) = \mathbf{h}(n) * \mathbf{c}(n)$ and has length $L_c + L_e - 1$. Here, the output $y(n)$ is of the form,

$$y(n) = x_f(n) + q(n)$$

where $x_f(n)$ and $q(n)$ are, respectively, the filtered input signal and output noise sequences given by the following.

$$x_f(n) = c_{\text{eff}}(n) * x(n)\,,\ q(n) = \mathbf{h}(n) * \boldsymbol{\eta}(n)$$

We wish to choose $\mathbf{H}(z)$ to shorten the effective channel to a length $L_d < L_c$. In other words, we wish to choose $\mathbf{H}(z)$ such that most of the substance of $c_{\text{eff}}(n)$ resides in a window $\mathcal{W}_\Delta \triangleq [\Delta, \Delta + L_d - 1]$, where Δ represents the delay of the desired shortened channel. Here we must have $0 \leq \Delta \leq L_c + L_e - L_d - 1$. To that end, we define the following responses.

$$\begin{aligned} c_{\text{des}}(n) &= w_\Delta(n) c_{\text{eff}}(n) && \text{(desired response)} \\ c_{\text{res}}(n) &= (1 - w_\Delta(n))\, c_{\text{eff}}(n) && \text{(residual response)} \end{aligned}$$

where $w_\Delta(n)$ is the following rectangular window function.

$$w_\Delta(n) = \begin{cases} 1\,, & \Delta \le n \le \Delta + L_d - 1 \\ 0\,, & \text{otherwise} \end{cases}$$

In this case, we can write $x_f(n)$ as,

$$x_f(n) = x_{\text{des}}(n) + x_{\text{res}}(n)$$

where we have,

$$x_{\text{des}}(n) = c_{\text{des}}(n) * x(n)\,,\; x_{\text{res}}(n) = c_{\text{res}}(n) * x(n)$$

With this decomposition, we wish to choose the coefficients of $\mathbf{H}(z)$ to accomplish the following goals.

- Minimize the ISI power of $x_{\text{res}}(n)$.
- Minimize the noise power of $q(n)$.

We wish to minimize both of these quantities with respect to the desired signal power of $x_{\text{des}}(n)$. To that end, we propose to choose $\mathbf{H}(z)$ to minimize the following objective function.

$$J \triangleq \frac{\alpha\sigma^2_{x_{\text{res}}} + (1-\alpha)\sigma^2_q}{\sigma^2_{x_{\text{des}}}}\,,\; 0 \le \alpha \le 1 \qquad (1)$$

Here, α is a tradeoff parameter between minimizing the ISI power of $x_{\text{res}}(n)$ and the noise power of $q(n)$. In practice, the delay parameter Δ should be varied over all admissible values and chosen so as to minimize J in (1). We now proceed to analyze J.

3. ANALYSIS OF THE OBJECTIVE FUNCTION J

Let us define the following vectors and matrices.

$$\begin{aligned} \mathbf{c}_{\text{eff}} &\triangleq \begin{bmatrix} c_{\text{eff}}(0) & c_{\text{eff}}(1) & \cdots & c_{\text{eff}}(L_c+L_e-2) \end{bmatrix} \\ \mathbf{h} &\triangleq \begin{bmatrix} \mathbf{h}(0) & \mathbf{h}(1) & \cdots & \mathbf{h}(L_e-1) \end{bmatrix} \end{aligned}$$

$$\mathbf{C} \triangleq \begin{bmatrix} \mathbf{c}(0) & \mathbf{c}(1) & \cdots & \mathbf{c}(L_c-1) & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{c}(0) & \mathbf{c}(1) & \cdots & \mathbf{c}(L_c-1) & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{c}(0) & \mathbf{c}(1) & \cdots & \mathbf{c}(L_c-1) \end{bmatrix}$$

$$\begin{aligned} \mathbf{W}_\Delta &\triangleq \begin{bmatrix} \mathbf{0}_\Delta & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{Ld} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0}_{L_c+L_e-L_d-1-\Delta} \end{bmatrix} \\ \overline{\mathbf{W}}_\Delta &\triangleq \mathbf{I}_{L_c+L_e-1} - \mathbf{W}_\Delta \end{aligned}$$

These quantities have the following sizes.

- $\mathbf{c}_{\text{eff}} : 1 \times (L_c + L_e - 1)$
- $\mathbf{h} : 1 \times KL_e$
- $\mathbf{C} : KL_e \times (L_c + L_e - 1)$
- $\mathbf{W}_\Delta, \overline{\mathbf{W}}_\Delta : (L_c + L_e - 1) \times (L_c + L_e - 1)$

Observe that $\mathbf{W}_\Delta \mathbf{W}_\Delta^\dagger = \mathbf{W}_\Delta$ and similarly $\overline{\mathbf{W}}_\Delta \overline{\mathbf{W}}_\Delta^\dagger = \overline{\mathbf{W}}_\Delta$. Also, by the convolution $c_{\text{eff}}(n) = \mathbf{h}(n) * \mathbf{c}(n)$, we have,

$$\mathbf{c}_{\text{eff}} = \mathbf{hC}$$

Now, as $x_{\text{des}}(n) = c_{\text{des}}(n) * x(n)$ and since $x(n)$ is white with variance σ^2_x, we have the following.

$$\begin{aligned} \sigma^2_{x_{\text{des}}} &= \sigma^2_x \sum_n |c_{\text{des}}(n)|^2 = \sigma^2_x \sum_{n \in \mathcal{W}_\Delta} |c_{\text{eff}}(n)|^2 \\ &= \sigma^2_x \left(\mathbf{c}_{\text{eff}} \mathbf{W}_\Delta\right) \left(\mathbf{c}_{\text{eff}} \mathbf{W}_\Delta\right)^\dagger = \sigma^2_x \mathbf{c}_{\text{eff}} \mathbf{W}_\Delta \mathbf{W}_\Delta^\dagger \mathbf{c}_{\text{eff}}^\dagger \\ &= \sigma^2_x \mathbf{hC} \mathbf{W}_\Delta \mathbf{W}_\Delta^\dagger \mathbf{C}^\dagger \mathbf{h}^\dagger = \sigma^2_x \mathbf{hC} \mathbf{W}_\Delta \mathbf{C}^\dagger \mathbf{h}^\dagger \end{aligned} \qquad (2)$$

Similarly, we have,

$$\sigma^2_{x_{\text{res}}} = \sigma^2_x \mathbf{hC} \overline{\mathbf{W}}_\Delta \mathbf{C}^\dagger \mathbf{h}^\dagger \qquad (3)$$

Since $q(n) = \mathbf{h}(n) * \boldsymbol{\eta}(n)$, we have,

$$\sigma^2_q = R_{qq}(0) = \sum_{m,n} \mathbf{h}(m) \mathbf{R}_{\boldsymbol{\eta\eta}}(n-m) \mathbf{h}^\dagger(n)$$

Defining the $KL_e \times KL_e$ matrix $\mathbf{R}_{\boldsymbol{\eta}}$ as follows,

$$\mathbf{R}_{\boldsymbol{\eta}} \triangleq \begin{bmatrix} \mathbf{R}_{\boldsymbol{\eta\eta}}(0) & \mathbf{R}_{\boldsymbol{\eta\eta}}(1) & \cdots & \mathbf{R}_{\boldsymbol{\eta\eta}}(L_e-1) \\ \mathbf{R}_{\boldsymbol{\eta\eta}}(-1) & \mathbf{R}_{\boldsymbol{\eta\eta}}(0) & \ddots & \vdots \\ \vdots & \ddots & \ddots & \mathbf{R}_{\boldsymbol{\eta\eta}}(1) \\ \mathbf{R}_{\boldsymbol{\eta\eta}}(-(L_e-1)) & \cdots & \mathbf{R}_{\boldsymbol{\eta\eta}}(-1) & \mathbf{R}_{\boldsymbol{\eta\eta}}(0) \end{bmatrix}$$

we can express σ^2_q as shown below.

$$\sigma^2_q = \mathbf{h} \mathbf{R}_{\boldsymbol{\eta}} \mathbf{h}^\dagger \qquad (4)$$

Combining (2), (3), and (4) in (1) yields the following.

$$\begin{aligned} J &= \frac{\alpha\sigma^2_x \mathbf{hC}\overline{\mathbf{W}}_\Delta \mathbf{C}^\dagger \mathbf{h}^\dagger + (1-\alpha)\mathbf{h}\mathbf{R}_{\boldsymbol{\eta}}\mathbf{h}^\dagger}{\sigma^2_x \mathbf{hC}\mathbf{W}_\Delta \mathbf{C}^\dagger \mathbf{h}^\dagger} \\ &= \frac{\mathbf{h}\left[\alpha \mathbf{C}\overline{\mathbf{W}}_\Delta \mathbf{C}^\dagger + (1-\alpha)\frac{1}{\sigma^2_x}\mathbf{R}_{\boldsymbol{\eta}}\right]\mathbf{h}^\dagger}{\mathbf{hC}\mathbf{W}_\Delta \mathbf{C}^\dagger \mathbf{h}^\dagger} \end{aligned} \qquad (5)$$

Now, assuming that the $KL_e \times KL_e$ matrix $\mathbf{A}_\Delta \triangleq \mathbf{C}\mathbf{W}_\Delta \mathbf{C}^\dagger$ has a full rank of KL_e, then it is positive definite. As such, there exists a nonsingular $KL_e \times KL_e$ matrix $\mathbf{G}_\Delta$ such that [5],

$$\mathbf{A}_\Delta = \mathbf{G}_\Delta^\dagger \mathbf{G}_\Delta \qquad (6)$$

The choice of $\mathbf{G}_\Delta$ is not unique here. One such $\mathbf{G}_\Delta$ is upper triangular and is called the *Cholesky decomposition* [5] of $\mathbf{A}_\Delta$. A comment regarding the nonsingularity of $\mathbf{A}_\Delta$ is in order here. As $\mathbf{A}_\Delta = \mathbf{C}\mathbf{W}_\Delta \mathbf{C}^\dagger$ and $\mathbf{W}_\Delta$ clearly has rank L_d, we have the following necessary conditions for $\mathbf{A}_\Delta$ to be nonsingular.

- $KL_e \le (L_c + L_e - 1) \Longleftrightarrow L_e \le \frac{L_c - 1}{K-1}$
- $KL_e \le L_d \Longleftrightarrow L_e \le \frac{L_d}{K}$

Note that both of these conditions put an *upper bound* on the length of our equalizer. Assuming for now that $\mathbf{A}_\Delta$ is indeed of full rank, then we can express the objective function J as a Rayleigh quotient [5] using the decomposition of (6). Defining the $KL_e \times 1$ column vector $\mathbf{v}_\Delta$ as follows,

$$\mathbf{v}_\Delta \triangleq \mathbf{G}_\Delta \mathbf{h}^\dagger \Longleftrightarrow \mathbf{h} = \mathbf{v}_\Delta^\dagger \left(\mathbf{G}_\Delta^{-1}\right)^\dagger$$

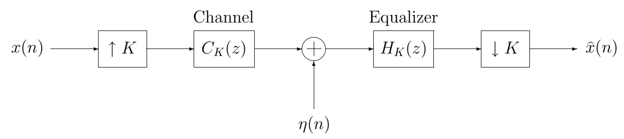

Fig. 2. Discrete-time model of a K-fold FSE.

we have from (5),

$$J = \frac{\mathbf{v}_\Delta^\dagger \left[\alpha\left(\mathbf{G}_\Delta^{-1}\right)^\dagger \mathbf{C}\overline{\mathbf{W}}_\Delta \mathbf{C}^\dagger \left(\mathbf{G}_\Delta^{-1}\right) + (1-\alpha)\frac{1}{\sigma_x^2}\left(\mathbf{G}_\Delta^{-1}\right)^\dagger \mathbf{R}_{\boldsymbol{\eta}} \left(\mathbf{G}_\Delta^{-1}\right)\right]\mathbf{v}_\Delta}{\mathbf{v}_\Delta^\dagger \mathbf{v}_\Delta}$$

More compactly, we have,

$$J = \frac{\mathbf{v}_\Delta^\dagger \mathbf{T}_\Delta \mathbf{v}_\Delta}{\mathbf{v}_\Delta^\dagger \mathbf{v}_\Delta}$$

where we have,

$$\mathbf{T}_\Delta \triangleq \alpha\left(\mathbf{G}_\Delta^{-1}\right)^\dagger \mathbf{C}\overline{\mathbf{W}}_\Delta \mathbf{C}^\dagger \left(\mathbf{G}_\Delta^{-1}\right) + (1-\alpha)\frac{1}{\sigma_x^2}\left(\mathbf{G}_\Delta^{-1}\right)^\dagger \mathbf{R}_{\boldsymbol{\eta}} \left(\mathbf{G}_\Delta^{-1}\right)$$

Since $\overline{\mathbf{W}}_\Delta$ and $\mathbf{R}_{\boldsymbol{\eta}}$ are Hermitian, so too is $\mathbf{T}_\Delta$. As such, we have thus expressed J as a Rayleigh quotient. By *Rayleigh's principle* [5], it follows that as $\mathbf{v}_\Delta$ varies over all nonzero vectors, the minimum value of the objective function J is $\lambda_{\Delta,\text{min}}$, where $\lambda_{\Delta,\text{min}}$ denotes the smallest eigenvalue of $\mathbf{T}_\Delta$. Furthermore, this minimum value is achieved if $\mathbf{v}_\Delta = \mathbf{v}_{\Delta,\text{min}}$, where $\mathbf{v}_{\Delta,\text{min}}$ denotes an eigenvector of $\mathbf{T}_\Delta$ corresponding to $\lambda_{\Delta,\text{min}}$. (More generally, the minimum value of J is achieved iff $\mathbf{v}_\Delta$ is in the eigenspace corresponding to $\lambda_{\Delta,\text{min}}$. However, for sake of clarity, we will ignore this scenario.) Hence, if J_{opt} and $\mathbf{h}_{\text{opt}}$ denote the optimum value of J and optimizing equalizer coefficients, respectively, we have,

$$\boxed{\begin{aligned} J_{\text{opt}} &= \lambda_{\Delta,\text{min}} \\ \mathbf{h}_{\text{opt}} &= \mathbf{v}_{\Delta,\text{min}}^\dagger \left(\mathbf{G}_\Delta^{-1}\right)^\dagger \end{aligned}}$$

The vector $\mathbf{h}_{\text{opt}}$ is called an *eigenfilter* [11, 8] as its elements are filter coefficients derived from an eigenvector of a matrix. We now show that the FSE is a special case of the structure in Figure 1.

4. RELATION TO FRACTIONALLY SPACED EQUALIZERS

The discrete-time model of a K-fold FSE is shown in Figure 2 [9, 10]. Here, $C_K(z)$ and $H_K(z)$ represent, respectively, a K-fold oversampled version of our original channel and equalizer. The noise process $\eta(n)$ is similarly a K-fold oversampled version of our original noise process. Consider the following polyphase decompositions [11] of $C_K(z)$ and $H_K(z)$ below.

$$C_K(z) = \sum_{k=0}^{K-1} z^k R_k(z^K)\,,\; H_K(z) = \sum_{k=0}^{K-1} z^{-k} E_k(z^K)$$

Using the *noble identities* [11], the structure in Figure 2 can be redrawn as in Figure 1 where we have,

$$[\mathbf{C}(z)]_{k,0} = R_k(z)\,,\; [\mathbf{H}(z)]_{0,k} = E_k(z)\,,\; [\boldsymbol{\eta}(n)]_{k,0} = \eta(Kn-k)$$

for $0 \le k \le K-1$. Because of this, if our goal is to choose the coefficients of the equalizer $H_K(z)$ to jointly minimize the ISI and noise power of the overall system, then these coefficients can be found using the eigenfilter approach of the previous section.

5. EXPERIMENTAL RESULTS

We now proceed to analyze how our design method compares with other known ones. One important figure of merit used to measure the performance of a TEQ in a DMT system is the maximum achievable bit rate. In a traditional DMT system, the subchannels are modeled as independent parallel Gaussian channels [7]. As a result, the number of bits b_k per real dimension to allocate in the k-th subchannel is given by the following [7].

$$b_k = \frac{1}{2}\log_2\left(1 + \frac{\text{SNR}_k}{\Gamma}\right)\,,\; 0 \le k \le N_{\text{DFT}} - 1$$

where N_{DFT} denotes the size of the Discrete Fourier Transform (DFT) used. Here, Γ is a "gap" quantity that depends on the coding and modulation format used as well as the desired probability of error. (For uncoded PAM and QAM constellations, $\Gamma = 9.8$ dB for a symbol error probability of 10^{-7} [7].) Also, SNR_k denotes the signal-to-noise ratio in the k-th subchannel and is given by [2],

$$\text{SNR}_k = \frac{\sigma_x^2 |C_{\text{des}}(e^{j\omega_k})|^2}{\sigma_x^2 |C_{\text{res}}(e^{j\omega_k})|^2 + S_{qq}(e^{j\omega_k})}\,,\; \omega_k = \frac{2\pi k}{N_{\text{DFT}}} \qquad (7)$$

where here, $L_d = N_{\text{CP}} + 1$ and N_{CP} is the cyclic prefix length.

In order to test our TEQ design method in a practical setting, such as the downstream link of a typical asymmetric digital subscriber line (ADSL) system, we make the following assumptions.

- Input signal consists of QAM symbols.
- Desired probability of error is 10^{-7}.
- Size of DFT is $N_{\text{DFT}} = 512$.
- Length of cyclic prefix is $N_{\text{CP}} = 32$.

As the input consists of two-dimensional QAM symbols, the number of bits to allocate in the k-th subchannel is given by,

$$b_k = \left\lfloor \log_2\left(1 + \frac{\text{SNR}_k}{\Gamma}\right)\right\rfloor\,,\; 0 \le k \le N_{\text{DFT}} - 1$$

with $\Gamma = 9.8$ dB and SNR_k as given in (7).

Data for the channel and noise was obtained from the `Matlab DMTTEQ Toolbox` [3]. We chose the following parameters.

- Input power is $\sigma_x^2 = 14$ dBm.
- Length of equalizer is $L_e = 16$ (for $K = 2$, $L_e = 8$).
- Carrier service area (CSA) loop # 1 was used as the channel ($L_c = 1{,}024$).
- Input noise consists of near-end crosstalk (NEXT) noise plus additive white noise with power density -110 dBm/Hz.
- Sampling frequency is $f_s = 2.208$ MHz.

Here we took the given channel $c_{\text{CSA}}(n)$ to be the K-fold oversampled channel for the FSE. As such, we took the *decimated* version

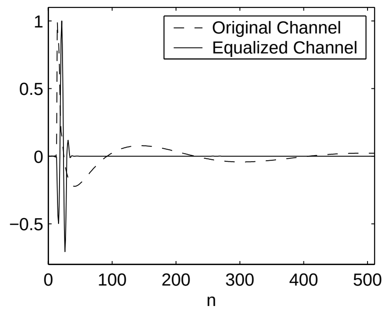

Fig. 3. Original and equalized channel impulse responses ($\alpha = 0.984$).

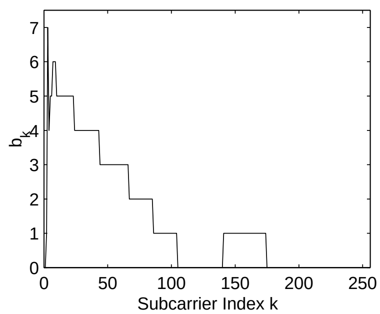

Fig. 4. Plot of b_k versus k using proposed method ($\alpha = 0.984$).

$c(n) = c_{\text{CSA}}(Kn)$ to be the observed channel for the SSE. For our simulations, we used $K = 2$. The bit rate R_b was calculated using,

$$R_b = \frac{f_s}{(N_{\text{DFT}} + N_{\text{CP}})} \sum_{k=0}^{N_{\text{DFT}}-1} b_k$$

Here, α in (1) was varied in order to obtain the largest possible R_b. The best SSE obtained was for $\alpha = 0.984$. The original and equalized channel impulse responses are shown in Figure 3. Also, in Figure 4, we plotted b_k as a function of the subcarrier index k for this equalizer. Only the values for $k = 0, \ldots, N_{\text{DFT}}/2$ are shown due to the mirror symmetry inherent in b_k due to the fact that the channel, equalizer, and noise are all real here.

In addition to our method, we also tested the following ones.

- Delay spread minimization by Schur and Speidel [6].
- Eigenapproach of Farhang-Boroujeny and Ding [4].
- Geometric SNR maximization by Al-Dhahir and Cioffi [1].

The observed bit rates for the TEQs tested here are shown in Table 1. From it, we can see that the SSE designed using our method performed better than all other SSEs considered. More interestingly, however, was the fact that the FSE designed using our method yielded the best results. The best FSE was obtained when $\alpha = 0.898$. This helps justify the future use of FSEs as TEQs for DMT systems. The advantage in bit rate may outweigh the overhead due to oversampling the output of the channel.

Method	R_b (Mb/s)
Eigenfilter Method - SSE ($\alpha = 0.984$)	2.841
Schur & Speidel [6]	2.362
Farhang-Boroujeny & Ding [4]	1.542
Al-Dhahir & Cioffi [1]	1.859
Eigenfilter Method - FSE ($\alpha = 0.898$)	3.515

Table 1. Observed bit rates (R_b) for different equalizer methods.

6. CONCLUDING REMARKS

In terms of achievable bit rate, the TEQs designed using our approach surpassed those by other methods. Of all TEQs considered, the FSE designed using our method performed the best. This helps justify using FSEs for DMT systems. Using the eigenfilter method for other objective functions is the subject of ongoing research.

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