Improvements in equalization of multiuser CDMA systems: oversampling and nonuniqueness

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Abstract

Major challenges in the design of new generation wireless systems are the suppression of multiuser interference (MUI) and inter-symbol interference (ISI) within a single user resulting from frequency selective propagation. Both of these problems were addressed successfully in the recent design of A Mutually-Orthogonal Usercode-Receiver (AMOUR) for CDMA systems. An attractive property of AMOUR is that it guarantees user separation and the existence of simple zero-forcing equalizers (ZFE) regardless of the channels. However, these ZFEs can amplify the noise significantly and in this paper we propose a method aimed at improving their performance. First, we note that oversampling at the receiver yields additional degrees of freedom in the ZFE design. This redundancy is then used to construct the solution that minimizes the noise power at the detector. The method is tested in computer simulations.[1]

1 Introduction

Multiuser interference (MUI) and inter-symbol interference (ISI) continue to top the list of factors limiting the performance of multiuser systems. MUI has traditionally been combated by the use of orthogonal spreading codes at the transmitter [4], however this orthogonality is often lost after the transmitted signals have passed through multipath channels. Furthermore, in the multiuser uplink scenario, exact equalization is possible only under certain conditions on the channel matrices. The alternative approach is to suppress MUI statistically, however this is often less desirable. Our approach is based on a recently developed method for user separation, called A Mutually-Orthogonal Usercode-Receiver (AMOUR) [1]–[2]. It aims at eliminating MUI deterministically while at the same time mitigating the undesired effects of multipath propagation for each user separately. One clear advantage of this over the previously known methods is that MUI elimination is achieved *irrespective of the channel nulls*; in addition to this, the existence of zero-forcing equalizers (ZFEs) is guaranteed *regardless of the channels*. (Both properties hold true as long as the maximum channel order L is known.) Zero forcing equalizers are rather attractive from the aspect of computational complexity, since their taps depend only on the channel realization. However, their performance varies significantly as the channel and the noise statistics change. They can amplify the noise at the receiver to the point where reliable communications become impossible.

In this work we propose an improvement of the basic AMOUR-CDMA system described in [1], achieved by signal *oversampling* at the receivers. This equalizer structure developed in Sec. 3 can be considered as a *fractionally spaced equalizer* (FSE) and thus we name the method Fractionally-Spaced AMOUR (FSAMOUR). The oversampling ratios are assumed to be integers, however, the method can be extended to include rational factors as shown in [5]. In systems with redundancy, such as this one, ZFEs are not unique, so that we can avoid the solutions which amplify the received noise. This leads to an improved performance of FSAMOUR systems. While the redundancy is present even in the traditional AMOUR case, in Sec. 4 we show that the improvement is usually much more pronounced when the method is used in combination with signal oversampling. Furthermore, we proceed to find the optimal zero-forcing solution.

2 AMOUR-CDMA systems

The structure in Fig. 1 describes the AMOUR-CDMA system for M users, i.e. M transmitters and M potential receivers. The upper part of the figure shows the mth transmitter followed by the uplink channel corresponding to the mth user and the lower part shows the receiver tuned to the user m. The symbol stream $s_m(n)$ is first blocked into a vector signal $\mathbf{s}_m(n)$ of length K. This signal is upsampled by $P > K$ and passed through a synthesis filterbank of *spreading codes* $\{C_{m,k}(z)\}_{k=0}^{K-1}$; thus each of the transmitters introduces *redundancy* in the amount of P/K. It will become clear that large values of K keep the overall bandwidth expansion moderate. Moreover, for large K, this ratio tends to M, i.e. the minimum value required in a system with M users.

The channels $H_m(z)$ are considered to be FIR of order $\leq L$. The mth receiver is functionally divided into three parts: filterbank $\{G_{m,j}(z)\}_{j=0}^{J-1}$ for MUI cancellation, block $\mathbf{V}_m^{-1}$ which is supposed to eliminate the effects of $\{C_{m,k}(z)\}$ and $\{G_{m,j}(z)\}$ on the desired signal $s_m(n)$, and the equalizer $\mathbf{\Gamma}_m$ aimed at reducing the ISI introduced by

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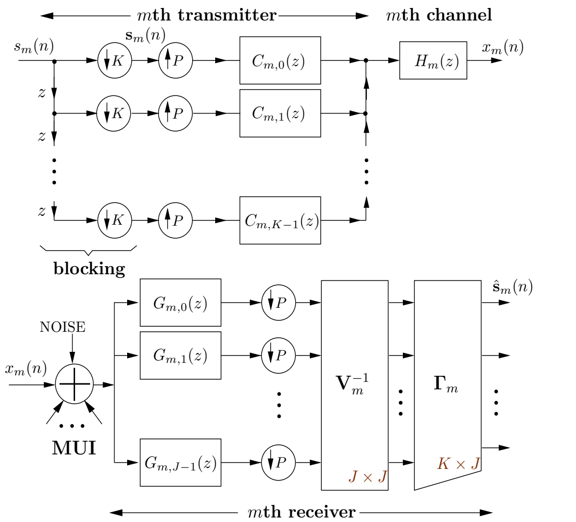

Figure 1: Discrete-time equivalent of a baseband AMOUR system.

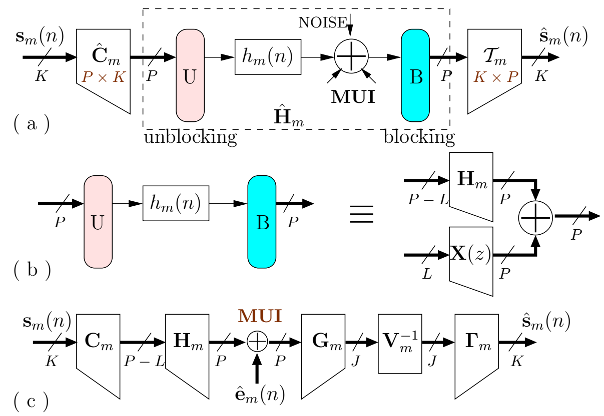

Figure 2: (a)-(c) Equivalent drawings of a symbol-spaced AMOUR system.

the multipath channel $H_m(z)$. Filters $G_{m,j}(z)$ are chosen to be FIR and are designed jointly with $\{C_{m,k}(z)\}$ to filter out the signals from the undesired users $\mu \neq m$. The choice of $\{C_{m,k}(z)\}$ and $\{G_{m,j}(z)\}$ is completely *independent* of the channels $H_m(z)$ and depends only on the maximum channel order L. Therefore, in this paper we assume that CSI is available only at the block-equalizers $\mathbf{\Gamma}_m$. If the channels are altogether unknown, some of the well-known *blind equalization* techniques can be incorporated at the receiver [2].

In the following we design each of the transmitter and receiver building blocks by rewriting them in a matrix form. The banks of filters $\{C_{m,k}(z)\}$ and $\{G_{m,j}(z)\}$ can be represented in terms of the corresponding $P \times K$ and $J \times P$ polyphase matrices $\hat{\mathbf{C}}_m$ and $\mathbf{G}_m$ respectively [3]. The (j,i)th element of $\mathbf{G}_m$ is given by $g_{m,j}(i)$ and the (i,k)th element of $\hat{\mathbf{C}}_m$ by $c_{m,k}(i)$. Note that the polyphase matrices $\hat{\mathbf{C}}_m$ and $\mathbf{G}_m$ become *constant* once we restrict the filters $C_{m,k}(z)$ and $G_{m,j}(z)$ to length P.

The system from Fig. 1 can now be redrawn as in Fig. 2(a), where the receiver block is defined as $\mathcal{T}_m = \mathbf{\Gamma}_m \mathbf{V}_m^{-1} \mathbf{G}_m$. The $P \times P$ block in Fig. 2(a) consisting of the signal unblocking, filtering through the mth channel and blocking can be equivalently described by the pseudo-circulant matrix $\hat{\mathbf{H}}_m = [\mathbf{H}_m \quad \mathbf{X}(z)]$; see Fig. 2(b). Here we denoted by $\mathbf{H}_m$ the $P \times (P-L)$ full banded lower triangular Toeplitz matrix with the first column given by $[h_m(0) \quad \cdots \quad h_m(L) \quad 0 \quad \cdots \quad 0]^T$, whereas $\mathbf{X}(z)$ is the $P \times L$ block that introduces the IBI. By choosing the last L samples of the spreading codes $\{C_{m,k}(z)\}$ to be zero, $\hat{\mathbf{C}}_m$ is of the form $\hat{\mathbf{C}}_m = [\mathbf{C}_m^T \quad \mathbf{0}^T]^T$ with the $L \times K$ zero-block positioned appropriately to eliminate the IBI block $\mathbf{X}(z)$, namely we have $\hat{\mathbf{H}}_m \hat{\mathbf{C}}_m = \mathbf{H}_m \mathbf{C}_m$. The IBI-free equivalent scheme is shown in Fig. 2(c), with the noise vector signal $\hat{\mathbf{e}}_m(n)$ obtained by blocking the noise from Fig. 2(a). Next we use the fact that full banded Toeplitz matrices can be diagonalized by Vandermonde matrices. Namely, let us choose

$$\mathbf{G}_m = \begin{bmatrix} 1 & \rho_{m,0}^{-1} & \cdots & \rho_{m,0}^{-P+1} \\ 1 & \rho_{m,1}^{-1} & \cdots & \rho_{m,1}^{-P+1} \\ \vdots & \vdots & & \vdots \\ 1 & \rho_{m,J-1}^{-1} & \cdots & \rho_{m,J-1}^{-P+1} \end{bmatrix}, \quad \text{for} \quad \rho_{m,j} \in \mathbb{C}, \tag{1}$$

denote by $\mathbf{\Theta}_m$ the first $P-L$ columns of $\mathbf{G}_m$ and define the diagonal matrix

$$\mathcal{H}_m(\boldsymbol{\rho}_m) \triangleq \text{diag}[H_m(\rho_{m,0}), H_m(\rho_{m,1}), \cdots, H_m(\rho_{m,J-1})], \tag{2}$$

with the argument defined as $\boldsymbol{\rho}_m \triangleq [\rho_{m,0} \ \ \rho_{m,1} \ \cdots \ \rho_{m,J-1}]$. For any $J \in \mathbb{N}$ and an arbitrary set of complex numbers $\{\rho_{m,j}\}_{j=0}^{J-1}$ the following holds

$$\mathbf{G}_m \mathbf{H}_m = \mathcal{H}_m(\boldsymbol{\rho}_m) \mathbf{\Theta}_m. \tag{3}$$

The choice of $\{\rho_{m,j}\}_{j=0}^{J-1}$ (which are also called *signature points*) is such that $\mathbf{G}_m$ eliminates MUI as explained next. It will become clear that any set of distinct signature points guarantees the channels are equalizable even if unknown.

Consider the interference from user $\mu \neq m$. From Fig. 2(c) it follows that the interfering signal $\mathbf{s}_\mu(n)$ passes through the concatenation of matrices

$$\mathbf{G}_m \mathbf{H}_\mu \mathbf{C}_\mu = \mathcal{H}_\mu(\boldsymbol{\rho}_m) \mathbf{\Theta}_m \mathbf{C}_\mu = \mathcal{H}_\mu(\boldsymbol{\rho}_m) \mathcal{C}_\mu(\boldsymbol{\rho}_m), \tag{4}$$

where the (j,k)th entry in the $J \times K$ matrix $\mathcal{C}_\mu(\boldsymbol{\rho}_m)$ is $C_{\mu,k}(\rho_{m,j})$. The first equality in (4) is a consequence of (3). From (4) we see that in order to eliminate MUI *regardless of the channels* it suffices to choose $\{\rho_{m,j}\}_{m,j=0}^{M-1,J-1}$ so that

$$C_{\mu,k}(\rho_{m,j}) = 0, \quad \forall m \neq \mu, \quad \forall k \in [0, K-1], \quad \forall j \in [0, J-1]. \tag{5}$$

In practice, the signature points $\rho_{m,j}$ are often chosen to be uniformly spaced on the unit circle

$$\rho_{m,l} = e^{j\frac{2\pi(m+lM)}{MJ}}, \quad 0 \le l \le J-1, \tag{6}$$

since this leads to FFT based AMOUR implementations having low complexity [1]. Equations (5) define $(M-1)J$ zeros of the polynomials $C_{m,k}(z)$. In addition to this, let $C_{m,k}(z)$ be such that

$$C_{m,k}(\rho_{m,j}) = A_m \rho_{m,j}^{-k}, \tag{7}$$

where the multipliers A_m introduce a simple power control for different users. At this point the total number of constraints for each of the spreading polynomials is equal to MJ. Recalling that the last L samples of spreading codes are fixed to be zero, the minimum spreading code length is given by $P = MJ + L$. Substituting (7) in (4) for $\mu = m$ and recalling (3) we have

$$\mathbf{G}_m\mathbf{H}_m\mathbf{C}_m = A_m \underbrace{\begin{bmatrix} 1 & \rho_{m,0}^{-1} & \cdots & \rho_{m,0}^{-J+1} \\ 1 & \rho_{m,1}^{-1} & \cdots & \rho_{m,1}^{-J+1} \\ \vdots & \vdots & & \vdots \\ 1 & \rho_{m,J-1}^{-1} & \cdots & \rho_{m,J-1}^{-J+1} \end{bmatrix}}_{\mathbf{V}_m} \bar{\mathbf{H}}_m, \tag{8}$$

where $\bar{\mathbf{H}}_m$ is the $J \times K$ north-west submatrix of $\mathbf{H}_m$.

In order to perform the channel equalization after MUI has been eliminated we need to invert the matrix product $\mathbf{V}_m\bar{\mathbf{H}}_m$ in (8), which in turn needs to be of sufficient rank. From (4) with $\mu = m$ we conclude that (8) can be further written as a product of a diagonal matrix $\mathcal{H}_m(\boldsymbol{\rho}_m)$ and a $J \times K$ Vandermonde matrix $\mathcal{C}_\mu(\boldsymbol{\rho}_m)$. The second matrix $\mathcal{C}_\mu(\boldsymbol{\rho}_m)$ is invertible as long as $\{\rho_{m,j}\}$ are distinct. The rank of $\mathcal{H}_m(\boldsymbol{\rho}_m)$ can drop by at most L, and this only if all the zeros of $H_m(z)$ occur at the signature points $\rho_{m,j}$. Thus, the sufficient condition for the invertibility of (8) regardless of the channel is that $J = K + L$.

The bandwidth expansion reduces with increasing K and when K tends to infinity the it becomes

$$\text{BW expansion} = \frac{P}{K} = [M(K+L)+L]/K \overset{K\to\infty}{\longrightarrow} M.$$

Since there are M simultaneous transmitters in the system, this is the minimum possible BW expansion per user.

From Fig. 2(c) it follows that (ignoring the noise)

$$\hat{\mathbf{s}}_m(n) = A_m\boldsymbol{\Gamma}_m\mathbf{V}_m^{-1}\mathbf{V}_m\bar{\mathbf{H}}_m\mathbf{s}_m(n) = A_m\boldsymbol{\Gamma}_m\bar{\mathbf{H}}_m\mathbf{s}_m(n). \tag{9}$$

Now, $\boldsymbol{\Gamma}_m$ can for example be chosen to eliminate ISI in the absence of noise and this would be a zero-forcing equalizer (ZFE). For more details on this and alternative equalizers, the reader is referred to [1]. In the following we consider the improvement of this conventional AMOUR system obtained by sampling the received continuous-time signal more densely than at the symbol-rate given by the transmitters.

3 AMOUR with oversampling

Fractionally-spaced equalizers (FSE) typically show an improvement in performance over SSEs at the expense of more computations per unit time required at the receiver. FSEs operate on a discrete signal obtained by sampling the received continuous-time signal q times faster than at symbol rate (thus the name fractionally-spaced). Our goal in this section is to introduce the benefits of FSEs in the ISI suppression, without violating the conditions for perfect MUI cancellation irrespective of the uplink channels. As will be clear shortly, this is entirely achieved by using the fractionally-spaced AMOUR (FSAMOUR) system, introduced herein.

Consider the continuous-time AMOUR system with FSE in Fig. 3(a). The information sequence $s_m(n)$ appears with symbol spacing PT/K, and the rate of the transmitted signal $u_m(n)$ is $1/T$. Before entering the mth uplink channel, it is converted into an analog signal and passed through a pulse shaping filter. The combined effect on $u_m(n)$ is called the *equivalent channel* and is denoted by $h_c(t)$. The received waveform $x_c(t)$, corrupted by noise and MUI is sampled at q times the rate at the output of the transmitter; here q is an integer greater than one. The sequence $x_m(n)$ with rate q/T enters the fractionally-spaced equalizer and the resulting signal is downsampled, so that the sequence $\hat{s}_m(n)$ at the decision device has exactly the same rate K/PT as the information sequence $s_m(n)$.

Now we derive the discrete-time equivalent of the oversampled system from Fig. 3(a). The received sequence $x_m(n)$ in the absence of noise and MUI is given by

$$x_m(n) = x_c(n\frac{T}{q}) = \sum_{k=-\infty}^{\infty} u_m(k)h_c(n\frac{T}{q} - kT). \tag{10}$$

By defining the discrete time sequence $h_m^{(q)}(n) \triangleq h_c(nT/q)$, which is actually the function $h_c(t)$ sampled q times more densely than at integers, we have

$$x_m(n) = \sum_{k=-\infty}^{\infty} u_m(k)h_m^{(q)}(n-kq). \tag{11}$$

This is shown in Fig. 3(b). Notice that although the discrete-time *equivalent structure* incorporates the upsampling by q at the output of the transmitters, this does not result in any bandwidth expansion, since the physical structure is still given in Fig. 3(a). For simplicity, in what follows we assume $q = 2$, however it is easy to show that a similar design procedure follows through for any integer q. First we redraw the structure in Fig. 3(b) as shown in Fig. 3(c), where $H_{m,0}(z)$ and $H_{m,1}(z)$ are the Type-1 polyphase components [3] of the oversampled filter $H_m^{(2)}(z)$. In other words $H_m^{(2)}(z) = H_{m,0}(z^2) + z^{-1}H_{m,1}(z^2)$. The proposed form of the equalizer with rate reduction in the FSAMOUR system is now shown in Fig. 4. In each of the branches of the equalizer the symbol rate is equal to $1/T$. Each of the corresponding receiver structures resemble that from Fig. 2(a). However, while the matrices $\mathbf{G}_m$ and $\mathbf{V}_m^{-1}$ are

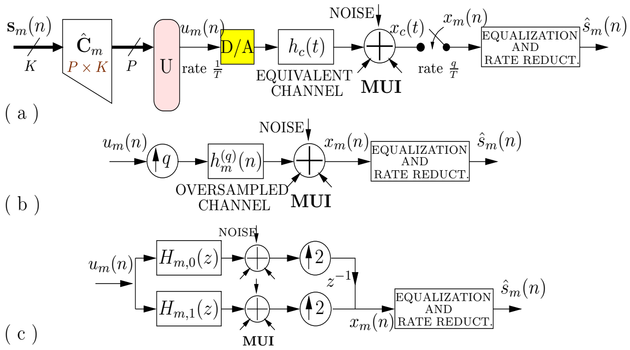

Figure 3: (a) Continuous-time model for the AMOUR system with oversampling. (b) Discrete-time equivalent drawing. (c) Polyphase representation for $q = 2$.

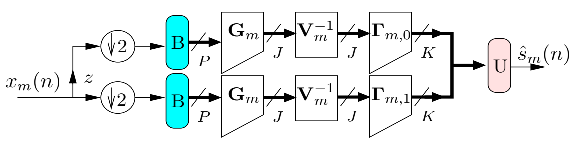

Figure 4: Proposed form of the equalizer with rate reduction.

kept the same as before, the matrices for ISI mitigation $\boldsymbol{\Gamma}_{m,i}$ are different in each branch and their outputs are combined, forming the information signal estimate $\hat{s}_m(n)$. Careful observation confirms that the output symbol rate is equal to K/PT, precisely as desired.

The complete FSAMOUR system is shown in Fig. 5(a). The effect of oversampling followed by the receiver structure with q branches is equivalent to receiving q copies of the transmitted signal, but after going through different multipath fading channels $H_{m,i}(z)$. This *temporal diversity* at the receiver is obviously beneficial for the equalization process as will be demonstrated. Notice that $H_{m,0}(z)$ is nothing but the original integer-sampled channel $H_m(z)$ and the orders of each of the subchannels $H_{m,i}(z)$ are bounded by L. Since MUI elimination in AMOUR systems does not depend on the uplink channels (as long as they are of order $\leq L$), it follows that the proposed FSAMOUR system will be MUI-free. Repeating the matrix manipulations similar to those demonstrated in Sec. 2, we conclude that the system can be redrawn as in Fig. 5(b). Lower triangular Toeplitz matrices $\bar{\mathbf{H}}_{m,i}$ here correspond to different polyphase components of the *oversampled* channel. Noise vectors $\mathbf{e}_i(n)$ are obtained by appropriately filtering the noise from Fig. 5(a).

The equalizer $\boldsymbol{\Gamma} = [\boldsymbol{\Gamma}_{m,0} \quad \boldsymbol{\Gamma}_{m,1}]$ can be constructed as a RAKE, zero-forcing or MMSE receiver corresponding to the transmitter $\bar{\mathbf{H}}_m = [\bar{\mathbf{H}}_{m,0}^T \quad \bar{\mathbf{H}}_{m,1}^T]^T$. The performance of *zero-forcing* solutions can be improved by noticing that

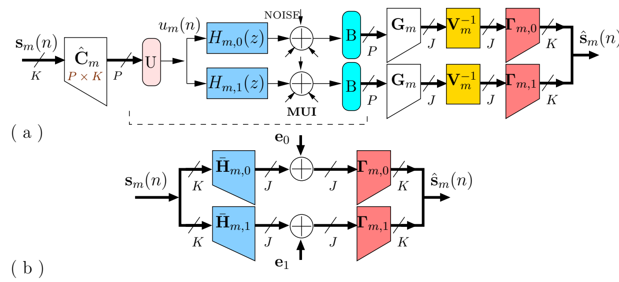

Figure 5: (a) A possible overall structure for the FSAMOUR system. (b) Simplified equivalent structure for ISI suppression.

left inverses of $\bar{\mathbf{H}}_m$ are not unique. In the following we derive the best ZFE for a given FSAMOUR system with the oversampling factor q.

4 Optimal FSAMOUR ZFE

Consider the system given in Fig. 6(a). It corresponds to the one in Fig. 5(b) with one difference, namely the block-equalizer is allowed to have memory. In the following we investigate the case of zero-forcing equalization, which corresponds to having $\hat{\mathbf{s}}_m(n) = \mathbf{s}_m(n)$ in the absence of noise. Obviously, this is achieved if and only if $\boldsymbol{\Gamma}_m(z)$ is a left inverse of $\bar{\mathbf{H}}_m$. Under the conditions on P and J described in Sec. 2 this inverse exists, but it is not unique. Our goal is to find the left inverse $\boldsymbol{\Gamma}_m(z)$ that minimizes the noise power at the output, i.e. minimizes the power of $\hat{\mathbf{s}}_m(n)$ given that $\mathbf{s}_m(n) = 0$. In order to solve the problem we consider the Smith form decomposition

$$\bar{\mathbf{H}}_m = \mathbf{U}_m \cdot \begin{bmatrix} \boldsymbol{\Sigma}_m \\ \mathbf{0} \end{bmatrix} \cdot \mathbf{V}_m, \qquad (12)$$

where $\mathbf{U}_m$ and $\mathbf{V}_m$ are $qJ \times qJ$ and $K \times K$ unitary matrices respectively, and $\boldsymbol{\Sigma}_m$ is a $K \times K$ diagonal matrix of singular values. From the aforementioned assumptions, it follows that $\boldsymbol{\Sigma}_m$ is invertible; the most general left inverse of $\bar{\mathbf{H}}_m$ is then given by $\boldsymbol{\Gamma}_m(z) = \mathbf{V}_m^\dagger \left[\boldsymbol{\Sigma}_m^{-1} \quad \mathbf{A}_m(z)\right] \mathbf{U}_m^\dagger$, where $\mathbf{A}_m(z)$ is an *arbitrary* $K \times (qJ - K)$ polynomial matrix and represents a handle on the degrees of freedom in the design of $\boldsymbol{\Gamma}_m(z)$. Defining the $K \times qJ$, $(qJ - K) \times qJ$ and $K \times (qJ - K)$ matrices $\mathbf{D}_0$, $\mathbf{D}_1$ and $\mathbf{B}_m(z)$ respectively as

$$\begin{bmatrix} \mathbf{D}_0 \\ \mathbf{D}_1 \end{bmatrix} \triangleq \mathbf{U}_m^\dagger, \quad \text{and} \quad \mathbf{B}_m(z) \triangleq \mathbf{V}_m^\dagger \cdot \mathbf{A}_m(z), \qquad (13)$$

we can re-write this as [see Fig. 6(b)]

$$\boldsymbol{\Gamma}_m(z) = \mathbf{V}_m^\dagger \boldsymbol{\Sigma}_m^{-1} \cdot \mathbf{D}_0 + \mathbf{B}_m(z) \cdot \mathbf{D}_1, \qquad (14)$$

and the design objective becomes that of finding $\mathbf{B}_m(z)$ that minimizes the noise power $E\{\hat{\mathbf{e}}_m^\dagger \hat{\mathbf{e}}_m\}/K$ at the output of Fig. 6(b). It is evident that the optimal $\mathbf{B}_m(z)$ in this context is nothing but a *linear estimator* of a vector

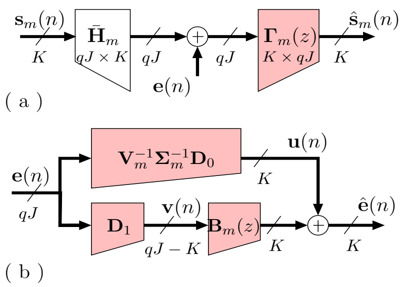

Figure 6: (a) Equivalent FSAMOUR system. (b) ZFE structure with noise input.

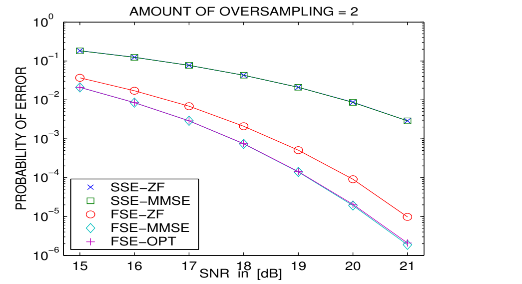

Figure 7: Probability of error as a function of SNR in AMOUR and FSAMOUR systems.

random process $\mathbf{u}(n)$ given $\mathbf{v}(n)$. The solution is well-known and it depends on the cross-correlation between the processes $\mathbf{u}(n)$ and $\mathbf{v}(n)$, as well as the autocorrelation of $\mathbf{v}(n)$. Rather than describing the general solution which has been presented in [5], we note that the memory in $\mathbf{B}_m(z)$, i.e. in $\boldsymbol{\Gamma}_m(z)$ does not buy anything in terms of performance unless the noise $\mathbf{e}(n)$ is strongly correlated across different blocks. For sufficiently large input block size qJ it is safe to assume that $\mathcal{R}_{\mathbf{ee}}(k) = \mathbf{0}$ for $k \neq 0$ and thus the optimal $\mathbf{B}_m(z)$ is a constant, namely

$$\mathbf{B}_m^{(\text{opt})}(z) = -\mathbf{V}_m^\dagger \boldsymbol{\Sigma}_m^{-1} \mathbf{D}_0 \mathcal{R}_{\mathbf{ee}}(0) \mathbf{D}_1^\dagger \left(\mathbf{D}_1 \mathcal{R}_{\mathbf{ee}}(0) \mathbf{D}_1^\dagger \right)^{-1}. \quad (15)$$

From (15) and (14) we get the optimal form of a ZFE

$$\boldsymbol{\Gamma}_m^{(\text{opt})} = \mathbf{V}_m^\dagger \boldsymbol{\Sigma}_m^{-1} \left[\mathbf{I}_K \;\; -\mathbf{D}_0 \mathcal{R}_{\mathbf{ee}}(0) \mathbf{D}_1^\dagger \left(\mathbf{D}_1 \mathcal{R}_{\mathbf{ee}}(0) \mathbf{D}_1^\dagger \right)^{-1} \right] \mathbf{U}_m^\dagger. \quad (16)$$

Another important special case occurs when the noise samples at the input of the receiver are i.i.d. It is important to notice here that $\mathbf{e}(n)$ in Figs. 5 and 6 is obtained by passing the input noise through a bank of q receiver front ends $\mathbf{V}_m^{-1}\mathbf{G}_m$. Therefore, the noise autocorrelation matrix $\mathcal{R}_{\mathbf{ee}}(0)$ is unlikely to be *equal* to a scaled identity. However, in systems with many users M when there is *no oversampling* and when $\mathbf{V}_m$ and $\mathbf{G}_m$ are chosen as in Sec. 2, it can be shown [5] that the approximation $\mathcal{R}_{\mathbf{ee}}(k) \approx \delta_k \cdot \sigma_k^2 \cdot \mathbf{I}$ is very accurate. In conclusion, we have that the optimal symbol-spaced ZFE in the systems with many users and uncorrelated noise reduces to

$$\boldsymbol{\Gamma}_m^{(\text{white noise})} = \mathbf{V}_m^\dagger \left[\boldsymbol{\Sigma}_m^{-1} \;\; \mathbf{0} \right] \mathbf{U}_m^\dagger. \quad (17)$$

It can be shown that (17) corresponds to a pseudo-inverse $\left(\bar{\mathbf{H}}_m^\dagger \bar{\mathbf{H}}_m\right)^{-1} \bar{\mathbf{H}}_m^\dagger$. In other words, if the channel noise is i.i.d. and there is no oversampling at the receiver, there is *nothing to be gained* by using the optimal solution. In contrast to this, with receiver oversampling the optimal solution remains (16) and the improvement in performance over the pseudo-inverse equalizer can be significant.

This point is demonstrated in Fig. 7 where we compare the performance of two symbol-spaced and three fractionally-spaced equalizers (using $q = 2$). Other system parameters were $K = 12$, $M = 4$, while J and P are chosen to be the minimum for the guaranteed existence of channel ZFEs as explained in Sec. 2. The simulation results are averaged over *thirty* independently chosen real random channels of order $L = 4$. The half-integer sampled channel impulse responses $h_m^{(2)}(n)$ were also chosen randomly, with the constraint that they coincide with AMOUR channels at integers. Under these assumptions we can see that the systems based on FSEs can significantly outperform the SSEs. Exploiting the redundancy of such solutions can result in further improvements (FSE-OPT). Finally, the performance of optimal FSEs as in (16) is almost identical to that of MMSE solutions. The latter depend on signal statistics, which makes them more complicated.

5 Concluding remarks

We propose an improvement of the AMOUR system called FSAMOUR that is based on oversampling the received signal. This leads to fractionally-spaced equalizers which are more robust in noisy environments. The implicit redundancy in the design of FSEs is exploited in Sec. 4, where we construct the optimal ZFEs in FSAMOUR. Performance improvements are demonstrated in simulations.

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