Fast and Robust Blind-Equalization Based On Cyclic Prefix

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***Abstract*-The cyclic prefix is commonly used in the context of frequency domain equalization in DMT channels. In this paper we observe that it can be used in more general contexts and show its advantages in blind equalization, especially for non minimum phase channels.**

I. Introduction

Techniques[1] for channel equalization based on redundant filter bank precoders have been introduced and well-studied in recent years [6], [10]. A major contribution [2] was the observation that the redundancy introduced by the precoder can be exploited to achieve **blind** equalization of FIR channels based only on second order statistics unlike classical methods based on fourth order moments [8]. Elegant theorems which make this practicable using computations based on received data are presented in [7] including procedures for channel identification, as well as for direct identification of input.

In this paper we show how the **cyclic prefix** system with DFT matrices, which is commonly employed in discrete multitone systems for twisted pair channels in telephone cables [1], [9], can actually be used for blind equalization of a much broader class of channels. One advantage of the proposed method is that besides FFT, there is very little computation involved (e.g., no need to identify annihilating eigenvectors and so forth as in [7]); the method is therefore very efficient.

If a channel has zeros in $|z| \geq 1$ then there are some problems associated with traditional equalization, blind or otherwise: the channel noise can get severly amplified. In this context a second advantage of the proposed method is that it does not require the channel to be minimum-phase. Equalization does not severely amplify noise as long as the zeros of the channel are not too close to the unit circle. The advantages of the proposed method are obtained at the expense of a slightly higher bandwidth expansion ratio compared to [7]. However, as the so-called block-length (a design parameter the user can choose) increases, this expansion becomes negligible as in [7].

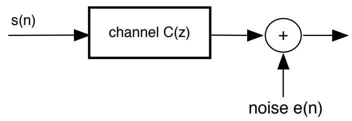

Figure 1. The FIR channel under consideration.

Throughout the sequel we shall assume that the channel is an Lth order FIR system

$$C(z) = \sum_{n=0}^{L} c(n) z^{-n} \qquad (1)$$

with additive noise $e(n)$ as shown in Fig. 1.

II. Redundancy in Channel Input

As a prelude to the main idea consider Fig. 2(a) which shows a symbol stream $s(n)$ (e.g., PAM symbols), divided into blocks of length M. Suppose we insert L zeros at the beginning of each block as shown in the figure. Then from measurements of output blocks we can recover the corresponding input blocks readily (see below). For a given symbol rate, the **zero-prefix** reduces the spacing between samples as demonstrated in Fig. 2(c). The bandwidth expansion factor

$$\gamma = \frac{M+L}{M}$$

represents the excess bandwidth required for this. By making M sufficiently large we can reduce γ but there are some compromises as we shall see. The zero-prefixed stream has blocks of length $L+M$ with the last M symbols in each block representing the original signal $s(n)$. The channel output can also be imagined as a succession of blocks of length $L+M$. Even though all the samples in a block can be nonzero, the last M samples of $y(n)$ in any block depend only on the samples $x(n)$ in the corresponding input block. The channel output depends on the input and the noise. Ignoring noise for a moment, we have for the kth block:

$$\begin{bmatrix} y(J_k) \\ y(J_k+1) \\ \vdots \\ y(J_k+M-1) \end{bmatrix} = \mathbf{C}_\Delta \begin{bmatrix} x(J_k) \\ x(J_k+1) \\ \vdots \\ x(J_k+M-1) \end{bmatrix} \qquad (2)$$

where $J_k = k(L+M)+L$ and

$$\mathbf{C}_\Delta = \begin{bmatrix} c(0) & 0 & \dots & 0 \\ c(1) & c(0) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ c(M-1) & c(M-2) & \dots & c(0) \end{bmatrix}$$

This is a lower triangular Toeplitz matrix representing causal convolution. Eq. (2) is valid for any $M > 0$ (if $M-1 > L$ then we have $c(M-1) = 0$, etc.).

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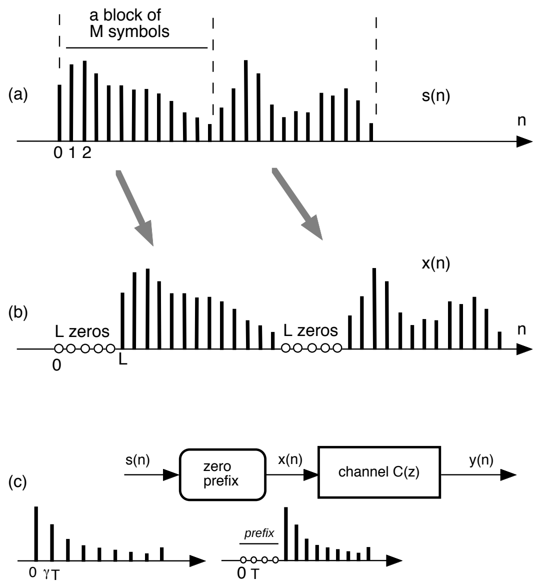

Figure 2. (a) A symbol stream, (b) zero-prefixed version, and (c) a schematic showing bandwidth expansion.

Assume[2] $c(0) \neq 0$. Then $\mathbf{C}_\Delta$ is nonsingular and its inverse is also lower triangular Toeplitz:

$$\mathbf{C}_\Delta^{-1} = \begin{bmatrix} h(0) & 0 & \dots & 0 \\ h(1) & h(0) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ h(M-1) & h(M-2) & \dots & h(0) \end{bmatrix} \quad (3)$$

where $h(n)$ are the first M coefficients of the inverse

$$\frac{1}{C(z)} = \sum_{n=0}^{\infty} h(n) z^{-n}. \quad (4)$$

Thus $\mathbf{C}_\Delta^{-1}$ can readily be found by long division, and the input symbols in each block obtained by inverting (2).

A. Noise amplification

In practice the channel adds noise, so $y(n)$ in Eq. (2) should be replaced with a **noisy version** $y(n)+e(n)$, and $x(n)$ is not recovered exactly. This noise could be severely amplified by the preceding inversion process if $C(z)$ has some zeros outside the unit circle. For, in this case the causal expansion (4) is unstable which means that the numbers $h(n)$ could be very large. For example if $C(z) = 1-2z^{-1}$ then $h(n) = 2^n, n \geq 0$. Such large values of n create *large amplification of the channel noise.*

B. Blind Identification

Assume next that the channel order L is known but $C(z)$ itself is unknown. Then it is still possible to recover the input stream from the output under some mild conditions. The fact that such **blind identification** can indeed be performed based on zero prefix is much less obvious, and is a direct consequence of the results first advanced in [2]. Techniques for implementing this, based on measurements of many successive output blocks (to satisfy rank conditions on certain matrices) can be found in [7]. These are based on the computation of annihilating eigenvectors of certain matrices formed from measurements of multiple blocks of the channel output. The performance of the method depends on the channel noise as well as the zeros of $C(z)$ for reasons similar to those described in Sec. A above.

First-principles method for blind identification. Now assume that we are a bit more generous in the matter of admitting redundancy. Thus instead of L zeros, suppose we insert $2L+1$ redundant samples as shown in Fig. 3. We call this the **impulse redundancy**. It has $2L$ zeros and an impulse in the middle. If this symbol stream is convolved with the Lth order channel $c(n)$, it is clear that the exact impulse response $c(n)$ appears at the output starting from the location of the redundant impulse in each block. Thus we can recover the channel exactly (without even a scale factor ambiguity!) and then invert it to recover the input $x(n)$. This is admittedly a trivial or "ultra simple" blind identification scheme but it differs from [7] only in the need for $2L+1$ rather than L redundant symbols. In return for this, the method is computationally and conceptually very simple.The bandwidth expansion factor $(2L+1+M)/M$ tends to unity for large M, as in [7]. Evidently the channel noise in practice contaminates the redundant impulse part as well, making the estimation of $c(n)$ noisy. However, the noise can be reduced by averaging the estimate over many blocks. This is analogous in principle (but certainly not in detail) to the accumulation of many blocks in [7] to satisfy certain rank conditions. Even assuming that the estimate of the channel is acceptable, the inverse matrix $\mathbf{C}_\Delta^{-1}$ could still amplify noise as in earlier methods, if the channel $C(z)$ is not minimum-phase. Finally note that any method which uses many blocks in the estimation naturally implies a correspondingly large latency.

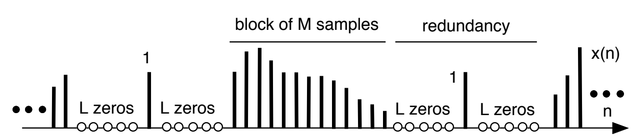

Figure 3. Impulse redundancy of length $2L+1$ added to every length-M block of the input symbol stream.

III. Cyclic-prefix redundancy

We now show that instead of using a zero prefix or impulse prefix we can also use a cyclic prefix and perform blind identification. One advantage of this method is that the equalization stage works even if the channel does not have minimum phase. We will see that it is sufficient (but not necessary) that the channel be free from unit circle zeros. Figure 4(a) again shows the input stream divided into blocks of length M. The L symbols at the end of each block are copied into the beginning of that block, to form the cyclic prefix (thin lines in Fig. 4(c)). This evidently assumes $L \leq M$. If we make the slightly stronger requirement that $L < M$, then the effect of the cyclic prefix is to replace the linear relation (2) with another linear relation where the matrix is **circulant** rather than lower triangular. Thus, the M input symbols $s(n)$ in the mth block are related to the last M output symbols $y(n)$ in the mth block as

$$\mathbf{y}(m) = \mathbf{C}\mathbf{s}(m) \quad (5)$$

[2]For this we can extract delays from $C(z)$ if necessary.

where

$$\mathbf{s}(m) = [\, s(mM) \quad s(mM+1) \quad \ldots \quad s(mM+M-1)\,]^T$$

$$\mathbf{y}(m) = [\, y(J_m) \quad y(J_m+1) \quad \ldots \quad y(J_m+M-1)\,]^T$$

with $J_m = m(L+M)+L$. The matrix $\mathbf{C}$ is circulant with the elements of the top row coming from the channel impulse response $c(n)$. For example when $L=3$ and $M=6$,

$$\mathbf{C} = \begin{pmatrix} c(0) & 0 & 0 & c(3) & c(2) & c(1) \\ c(1) & c(0) & 0 & 0 & c(3) & c(2) \\ c(2) & c(1) & c(0) & 0 & 0 & c(3) \\ c(3) & c(2) & c(1) & c(0) & 0 & 0 \\ 0 & c(3) & c(2) & c(1) & c(0) & 0 \\ 0 & 0 & c(3) & c(2) & c(1) & c(0) \end{pmatrix}$$

If the channel is known, we can perform the equalization by inverting (5) assuming $\mathbf{C}$ is nonsingular. The eigenvalues of the $M \times M$ circulant are equal to the DFT coefficients of the top row [4]. Since the top row has the channel coefficients in reversed order, these eigenvalues are

$$\eta(k) = \sum_{n=0}^{M-1} c(n) W^{-nk} = C(e^{-j2\pi k/M})$$

where $W = e^{-j2\pi/M}$ and $C(e^{j\omega})$ represents the channel frequency response. Thus $\eta(k)$ are obtained by sampling $C(e^{j\omega})$ uniformly at M frequencies. Note that $c(n)=0$ for $L<n<M$.

The circulant matrix $\mathbf{C}$ can be diagonalized with the DFT matrix [4]. More specifically we have

$$\mathbf{C} = \mathbf{W}^{-1}\mathbf{\Lambda}_c\mathbf{W}$$

where $\mathbf{W}$ is the $M \times M$ DFT matrix and

$$\mathbf{\Lambda}_c = \begin{bmatrix} C_M[0] & 0 & 0 & \ldots & 0 \\ 0 & C_M[1] & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots & \\ 0 & 0 & 0 & \ldots & C_M[M-1] \end{bmatrix}$$

where $C_M[k] = \sum_{n=0}^{L} c(n) W^{nk} = M$-point DFT of $c(n)$. Note that $C_M[k]$ is a permuted version of the eigenvalues $\eta(k)$. Thus the implementation of the communication system with cyclic prefix can be represented as shown in Fig. 5. The box labelled "blocking" is a serial to parallel converter (and "unblocking" converts from parallel to serial). The diagonal elements of $[\mathbf{\Lambda}_c]^{-1}$ are $1/C_M[k]$, and can be regarded as a set of DFT-domain equalizers.

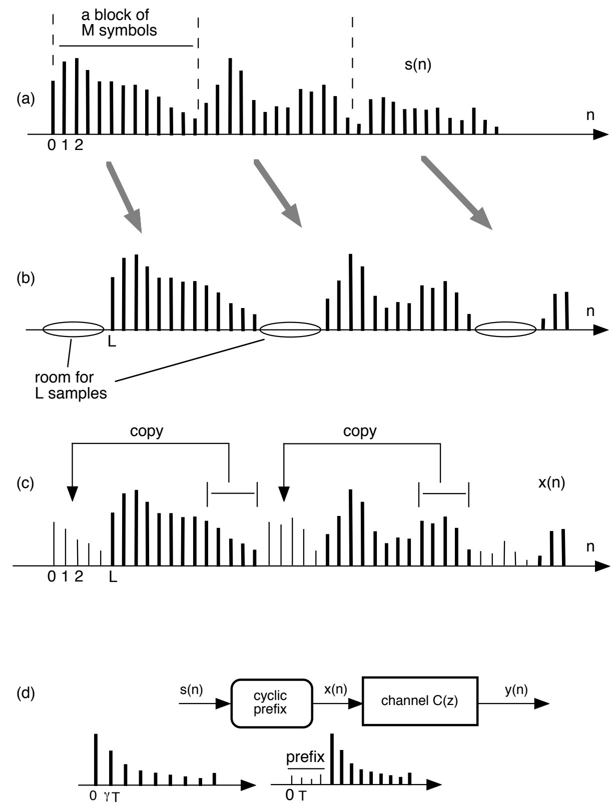

Figure 4. (a) Input symbol stream, (b)–(c) explanation of how cyclic prefix is inserted, and (d) block diagram.

Figure 5 of course ignores channel noise which in practice can be amplified by $[\mathbf{\Lambda}_c]^{-1}$. We shall address this later. Since $\mathbf{y}(m) = \mathbf{C}\mathbf{s}(m)$ according to Eq. (5), we can draw a schematic version of Fig. 5 as shown in Fig. 6(a). As $\mathbf{W}^{-1}$ is the inverse of $\mathbf{\Lambda}_c^{-1}\mathbf{W}\mathbf{C}$, we can redraw the system as in Fig. 6(b). In the first version the receiver has all the complexity whereas in the second version the IDFT is done at the transmitter, as in DMT systems. If the channel is known, then we can move $\mathbf{\Lambda}_c^{-1}$ and $\mathbf{W}$ to the transmitter end, yielding a useful configuration for cases where the receiver has to be the simplest. We can choose M to be a power of two and implement $\mathbf{W}$ and $\mathbf{W}^{-1}$ using radix-2 FFT.

Non minimum-phase channels. If the channel has no zeros on the unit circle, then $\mathbf{C}$ is nonsingular and we can invert (5) to obtain the input symbol stream. Once again the presence of noise makes everything imperfect as in other methods. If $C(z)$ has zeros close to the unit circle, then some of the $C_M[k]$ could be very small, and $\mathbf{C}$ becomes ill condition, thus amplifying the noise like the earlier methods described in Sec. II. However, unlike the earlier methods, the presence of zeros outside the unit circle does not create a problem because only the samples of $C(e^{j\omega})$ matter in the inversion. This appears to be an important advantage. For the hypothetical example where $C(z) = 1 - 2z^{-1}$ the methods described in Sec. II can yield a large noise gain because the coefficients of $\mathbf{C}_\Delta^{-1}$ are large. However in the cyclic prefix method, the DFT coefficients are bounded as

$$|C_M[k]| = |1 - 2e^{-j2\pi k/N}| \geq 1$$

so that the diagonal elements $1/C_M[k]$ of the equalizer $\mathbf{\Lambda}_c^{-1}$ in Fig. 6(b) do not amplify noise!

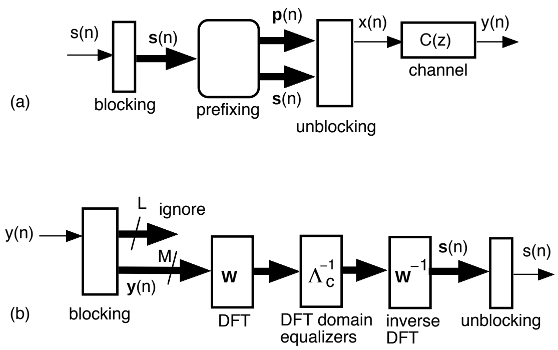

Figure 5. Block diagram description of the system based on cyclic prefix. (a) Transmitter, and (b) receiver.

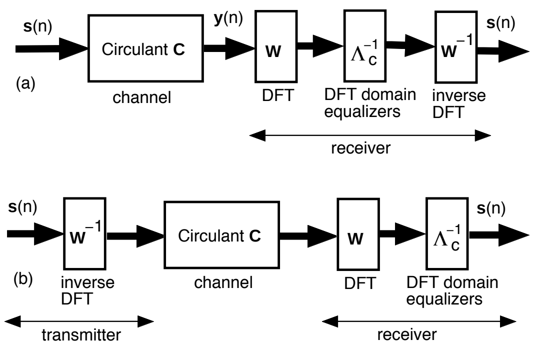

Figure 6. (a) A simplified schematic of the cyclic prefix system, and (b) a practically useful rearrangement similar to the conventional DMT system.

IV. Blind identification with cyclic-prefix

When the channel is unknown, the cyclic prefix method has to be further modified to allow blind identification. Thus consider the simple schematic of Fig. 6(b). Recall that this figure was obtained by introducing the length-L cyclic prefix at the beginning of each length-M block of $s(n)$. The vector $\mathbf{s}(n)$ represents the M samples in the nth block. Imagine that we introduce a further redundancy of $L+1$ samples into $\mathbf{s}(n)$. For example let $N = M + L + 1$ and define the N-vector

$$\mathbf{t}(n) = \begin{bmatrix} \mathbf{r}(n) \\ \mathbf{s}(n) \end{bmatrix}$$

where $\mathbf{r}(n)$ is a known vector of $L+1$ *nonzero samples*. Figure 6(b) is now modified into Fig. 7. Here the subscript N indicates the size of the matrices and distinguishes them from Fig. 6(b). Thus $\mathbf{C}_N$ is an $N \times N$ circulant, $\mathbf{W}_N$ represents the $N \times N$ DFT matrix, and the diagonal elements of $\mathbf{\Lambda}_{cN}^{-1}$ are $1/C_N[k]$ where

$$C_N[k] = \sum_{n=0}^{L} c(n) e^{-j2\pi kn/N}.$$

Fig. 7 represents an identity system as before. Since the channel is unknown we cannot insert the matrix $\mathbf{\Lambda}_{cN}^{-1}$ yet, but we can measure the signal $\mathbf{v}(n)$ at the receiver. The top $L+1$ components of $\mathbf{v}(n)$ represent the known vector $\mathbf{r}(n)$ scaled by $C_N[0], C_N[1], \ldots, C_N[L]$. Since $\mathbf{r}(n)$ has known nonzero samples, we can therefore identify $C_N[0], C_N[1], \ldots C_N[L]$. These are related to the channel $c(0), c(1), \ldots c(L)$ by the top/left $(L+1) \times (L+1)$ submatrix $\mathbf{W}_{sub}$ of the $N \times N$ DFT matrix. This submatrix is a nonsingular Vandermonde matrix. The channel $c(0), c(1), \ldots c(L)$ can therefore be identified.

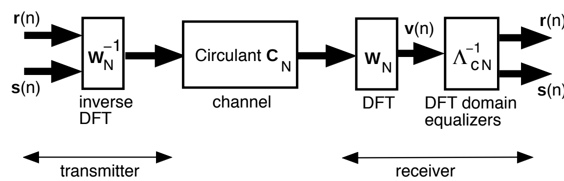

Figure 7. The cyclic prefix system with further redundancy for blind identification.

In practice $\mathbf{v}(n)$ has an additive noise component coming from the channel noise. As in other practical techniques [7] we can average the estimated $C_N[k]$ over a large number of output blocks, the compromise being the latency involved. Also for channels that vary in time, the rate at which we can update the blind estimation is compromised. Notice finally that blind identification has been possible only at the expense of the additional redundancy of $L+1$ samples. The complete system therefore has a higher bandwidth expansion factor $(M + 2L + 1)/M$. However, as in other methods, this approaches unity as M grows. Simulations (Sec. V) show that if we fix the bandwidth expansion ratio in the cyclic prefix method to be the same as for the method proposed in [7], then the former still performs better in many examples.

The redundancy $\mathbf{r}(n)$ need not necessarily be inserted at the beginning of the block. In fact for large M the submatrix $\mathbf{W}_{sub}$ can get ill conditioned because its columns "get closer". This results in undesirable amplification of channel noise. If we spread out the $L+1$ rendundant symbols more evenly this problem is less severe. If $L+1$ is a factor of N we can place the redundant symbols such that the submatrix to be inverted is itself the size-$(L+1)$ DFT matrix which is unitary up to scale.

V. Examples

We now consider an example where the channel $c(n)$ has order $L = 8$ and impulse response coefficients $1.0, 0.2, 0.5825, 0.25, 0.0892, -0.3463, -0.2886, 0.2757, 0.4$. This has four complex conjugate pairs of zeros, and the magnitudes are

$$1.092,\ 0.815,\ 0.927,\ \text{and}\ 0.767,$$

so there is one pair outside the unit circle. The quantity M was chosen as $M = 119$ so that the DFT sizes in Fig. 7 are $M+L+1 = 128$ which can be implemented with an FFT. The extra redundant symbols for blind identification were taken as $r(i) = 1$ for all i, and were distributed uniformly so that $\mathbf{W}_{sub}$ is a 9×9 DFT matrix.

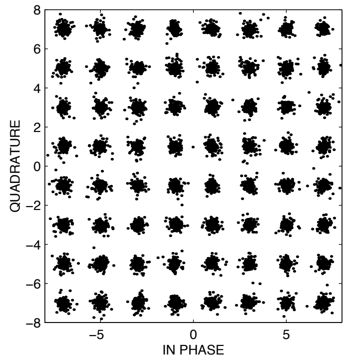

Figure 8. Result of blind identification (cyclic prefix).

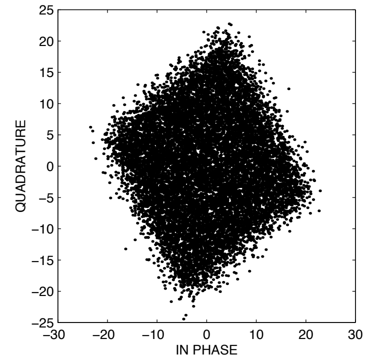

Figure 9. Result of blind identification based on [7].

The above channel was assumed unknown, and estimated using the method described in Sec. IV. The result of channel measurements in thirty successive blocks were averaged, and used in the detection process. With a 64-QAM constellation for the input $s(n)$, and an SNR of 21.5 dB at the receiver input in Fig. 7, the result of equalization after blind identification is as shown in Fig. 8. This corresponds to an average probability of error 6×10^{-5}. Notice that the bandwidth expansion factor was $(M + 2L + 1)/M = 136/119 = 8/7$. We tried to identify the same channel using the "direct blind identification method" of [7] for the same SNR and the same constellation. The block length was chosen as $M = 56$ so that the bandwidth expansion factor $(56 + 8)/56 = 8/7$ is the same as in our method. In this example the identification using the method of [7] was not successful and gave an average error probability of 0.3692 (see Fig. 9). It is possible that the method of [7] will work better if we use zero prefixes at other positions in each block rather than just at the beginning, as stated by Eq. (30) in [7]. This has not been attempted here.

VI. Concluding remarks

We have shown that the cyclic prefix method, which is traditionally used in DMT systems for channel equalization, can be modified readily for blind equalization of more general channels. Compared to the recent method of [7] this method requires a little more redundancy but it is computationally and conceptually much simpler. It also offers a smaller noise gain for channels with zeros outside the unit circle. The method also allows *frequency domain allocation* of power as in traditional DMT, but this was not exploited in Sec. V. Notice finally that a scale factor ambiguity in the channel, if complex, can rotate QAM consellations, which is undesirable (e.g., see Fig. 9). The blind identification method of Sec. IV does not have such ambiguities.

The idea of inserting redundancies into the blocks of the input is common to all the methods discussed in this paper, including those in [2] and [7]. For the case where the channel is unknown this allows blind identification. In fact, these methods can also be viewed as generalizations of the conventional method of sending training sequences periodically, once per block of M samples. The general theme in all these methods is to indirectly incorporate training sequences in a sophisticated and efficient way.

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