On the Capacity of Wireless Erasure Networks

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Abstract — **We determine the capacity of a certain class of wireless erasure relay networks. We first find a suitable definition for the "cut-capacity" of erasure networks with broadcast at transmission and no interference at reception. With this definition, a max-flow min-cut capacity result holds for the capacity of these networks.**

I. Introduction

For wireline networks, the max-flow min-cut result gives the network capacity between a single source and a single destination as well as in some multicast scenarios [1]. In this paper we consider a certain class of erasure wireless networks and show a max-flow min-cut capacity result for it. A detailed proof as well as extensions to some multicast scenarios can be found in [2].

II. Model

We model the network by a directed acyclic graph $G = (V, E)$. Each edge $(v_i, v_j) \in E$ represents a memoryless erasure channel from v_i to v_j with erasure probability $\epsilon_{i,j}$ associated with it. All channels are assumed independent and operate without delay.

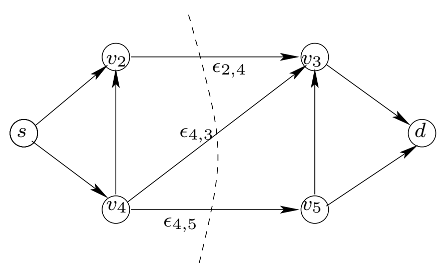

Figure 1: Example of a network. The capacity of the cut marked with a dotted line is $1 - \epsilon_{2,4} + 1 - \epsilon_{4,3}\epsilon_{4,5}$

Let $s = v_1 \in V$ be the source node that wishes to transmit a message to $d = v_{|V|}(\neq s) \in V$ which is the destination node. The other nodes simply have to aid this communication.

We incorporate broadcast in our network by insisting that vertex v_i transmit the same symbol on all outbound edges. This implies that v_5 must transmit the same message on edges (v_5, v_3) and (v_5, v_6). For reception, we assume that v_i receives the symbols from each incoming edge without interference. This means that v_2 receives messages from links (v_1, v_2) and (v_4, v_2) without their acting as interference for each other.

Finally, we assume that the decoder d knows the exact erasure pattern that occurred on each link of the network. This assumption requires a serious overhead of data transmission for a regular bit erasure channel. However, if we assume that the links are packet erasure channels, this assumption is reasonable for very long packets.

III. Definitions

An $s - d$ *cut* is defined as a partition of the vertex set V into two subsets V_s and $V_d = V - V_s$ such that $s \in V_s$ and $d \in V_d$. Clearly, an $s - d$ cut is determined simply by V_s. For the $s - d$ cut given by V_s, let the *cutset* $E(V_s)$ be the set of edges defined below

$$E(V_s) = \{(v_i, v_j) | (v_i, v_j) \in E, v_i \in V_s, v_j \in V_d\}$$

Denote by $W(V_s)$ the *value* of an $s-d$ cut given by V_s. We define $W(V_s)$ below.

$$W(V_s) = \sum_{i:(v_i,v_j)\in E(V_s)} \left(1 - \prod_{j:(v_i,v_j)\in E(V_s)} \epsilon_{i,j}\right) \qquad (1)$$

Consider this definition of the value of the cut or the "cut-capacity". In the wireline case, the value of the cut is simply the sum of the capacities of each edge in $E(V_s)$. Since our system model incorporates broadcast the cut-capacity is the sum of the capacities of each broadcast system that operates across the cut. This gives the outer summation in the definition of $W(V_s)$ above. For the capacity of each broadcast system that operates across the cut, assume that the receiver nodes within that system get to co-operate and hence have an effective erasure probability ϵ_{eff} equal to the product term in (1). Hence the capacity of each broadcast system is $(1 - \epsilon_{\text{eff}})$ which gives the quantity inside the summation.

In Fig. (1), the cut given by $V_s = \{s, v_2, v_4\}$ is marked with a dotted line. We have $E(V_s) = \{(v_2, v_3), (v_4, v_3), (v_4, v_5)\}$. This consists of the broadcast system emanating from v_4, viz., $\{(v_4, v_3), (v_4, v_5)\}$ and the (degenerate) broadcast system emanating from v_2, viz., $\{(v_2, v_3)\}$. The capacity of the former is $1 - \epsilon_{4,3}\epsilon_{4,5}$ and that of the latter is $1 - \epsilon_{2,4}$. Hence $W(V_s) = 1 - \epsilon_{4,3}\epsilon_{4,5} + 1 - \epsilon_{2,4}$.

IV. Main Result

We can then prove the following max-flow min-cut theorem.

Theorem 1. *The capacity of the erasure relay network described above is given by the value of the cut with minimum value.*

$$C = \min_{V_s} W(V_s)$$

where V_s determines an $s - d$ cut.

References

[1] R. Ahlswede, N. Cai, S.-Y. R. Li, R. W. Yeung, "Network information flow," *IEEE Trans. Info. Theory*, vol. 46, pp. 1204–1216, 2000.

[2] A. F. Dana, R. Gowaikar, R. Palanki, B. Hassibi, M. Effros, "On the Capacity of Wireless Erasure Broadcast Networks," *to be submitted to IEEE Trans. Info. Theory*

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