Geometry Seminar
Tuesday March 21 in room 613 WWH at 6:15 P.M

The rigidity of Hamming spaces, detecting of counterfeit coins and the Mastermind game

Grigori Kabatianski
DIMACS and Institute for Problems of Information Transmission, Moscow

Abstract

We call a set B a base of a metric space L if every point of L is uniquely determined by its distances to the points of B. The minimal possible number of points of a base we call the rigidity of the metric space. This notion was introduced in 1963 by P. Erdős and A. Renyi for solving the following weighing problem: what is the minimal number of weighings on an accurate scale to determine all false coins in a set of n coins. It is easy to see that the minimal number of queries with deterministic strategy differs by not more than one from the rigidity $r(2,n)$ of the binary n-dimensional Hamming space. The rigidity of the q-ary n-dimensional Hamming space is, in essence, the same as the minimal number of questions in the Mastermind game with deterministic strategy.

It is known that $r(2,n)$ is asymptotically equal to $2n/\log_2 n$ (B. Lindstrom 1964-66). For the non-binary case only upper and lower bounds on $r(q,n)$ are known (V.Chvatal,1983). In this talk we prove that for the ternary and quaternary cases the answer has the same

form as for the binary case with the corresponding replacment of the base of log, and make improvements for both types of bounds for $q > 4$.