ABSTRACT
We study the performance of a DS-CDMA blind multiuser detector in multipath Rayleigh fading channels for high data rates where the delay spread can be larger than one symbol duration. We propose blind recursive minimum output variance detectors for multipath channels with an additional constraint on the energy of the filter tap weights. This constraint prevents the cancellation of the desired signal in case the estimated subspace of the desired signal suffers from a mismatch. Simulations show that in cases of high Signal to Interference and Noise Ratios (SINR) as well as in cases of severe Inter-Symbol Interference (ISI), our proposed detector is superior to previously proposed detectors for multipath channels.

1. INTRODUCTION
In Direct Sequence Code Division Multiple Access (DS-CDMA) systems, each user is assigned a spreading code. All users transmit simultaneously in the same frequency band. Conventional detectors implemented in current systems simply correlate the received signal with the desired user’s signature and treat the Multiple Access Interference (MAI) due to other users as noise. Multiuser Detection (MUD) aims to suppress the interference due to MAI in order to increase the capacity of current systems and to avoid the need for strict power control. Next generation systems will support high data rates for multimedia applications. In such systems, the delay spread can be longer than the duration of one symbol, and the Inter-Symbol Interference (ISI) will be no longer negligible.

A blind multiuser detector based on minimizing the Minimum Output Energy (MOE) was proposed in [1] for an Additive White Gaussian Noise (AWGN) channel. It was borrowed from the theory of adaptive arrays and is also known as the Linear Constrained Minimum Variance receiver. It was shown that this detector could converge to the Minimum Mean Square Error (MMSE) solution without a training sequence, requiring only the knowledge of the desired user’s spreading sequence. The minimum variance receiver was modified to operate in multipath channels by several researchers as in [2]. Adaptive algorithms for its implementation along with joint channel estimation were proposed in [3]. The order of the channel (the maximum delay spread) is assumed known. This algorithm showed good performance for channels with small multipath delays.

In this paper, this algorithm is modified for cases where the order of the channel is underestimated or the delay spread is larger than one symbol duration. In such cases, the estimated signal subspace in which the desired signal was assumed to lie would suffer from a mismatch. Our proposed algorithm also showed better performance in channels with small multipath delays and high Signal to Interference Ratios (SINR).

2. SYSTEM MODEL
Consider a multiuser DS-CDMA system with $K$ users and $N$ chips per symbol transmitting asynchronously in a multipath Rayleigh fading channel. The $k_{th}$ user’s spreading code is given by $s_k = [s_k(0),...,s_k(N-1)]^T$ and is normalized such
that $s_k^T s_k = 1$. $N = T / T_c$ denotes the spreading gain, where $T$ and $T_c$ are the continuous time symbol and chip duration respectively. The received signal is processed by chip matched filtering and taking $U$ samples per chip. The baseband representation of the signal at the $n_{th}$ symbol interval is

$$\begin{align*}
r(n) = \sum_{m=-\infty}^{\infty} r_k(n) + z(n),
\end{align*}$$

where $z(n)$ is an AWGN with zero mean and variance $\sigma_z^2$. The signal due to user $k$ is

$$\begin{align*}
r_k(n) = \sum_{m=-\infty}^{\infty} \sqrt{P_k} d_i(m) h_i(n-mNU),
\end{align*}$$

where $d_i(n) = a_i(n) + j b_i(n)$ is the $n_{th}$ QPSK symbol of the $k_{th}$ user, $P_k$ is the received power of the $k_{th}$ user. The signature of user $k$ is

$$\begin{align*}
h_i(n) = \sum_{n=-\infty}^{\infty} g_l(n) s_l(n-mU - \Delta_i U),
\end{align*}$$

where $\tau_i = \Delta_i T_c$, $\tau_i \in [0, T]$ is the $k_{th}$ user’s delay, $g_i(n) = g_i(\tau_i)|_{\tau_i \in (T_m/2)}$ is the sampled multipath channel impulse response, where $L$ is the number of paths, $g_i(n) = \sum_{\tau = 0}^{L} \alpha_{\tau, i} \delta(t - \tau_s),$, and $\tau_s \in [0, T_m)$ is the delay of the $l_{th}$ path of the $k_{th}$ user, where $T_m$ is the delay spread. $g_i(n)$ is of maximum order $q$ where $q \in \left[ T_m / T_c \right]$ and $\left[ \right]$ denotes upper integer. The complex gain and power of the $l_{th}$ path of the $k_{th}$ user are $\alpha_{\tau, i}$ and $\sigma_{\tau, i}^2 = E[|\alpha_{\tau, i}|^2]$ respectively. The fading channel coefficients are complex Gaussian random variables with zero mean and normalized variance such that $\sum_{l=0}^{L} E[|\alpha_{\tau, i}(n)|^2] = 1 \ \forall k, l$. The fading process for each path is taken to be statistically independent of other paths as $E(\alpha_{\tau, i}(n) \alpha_{\tau, j}(n)^*) = \sigma_{\tau, i}^2 \delta_{k, k} \delta_{l, m}$, where $\delta$ is the discrete Kronecker delta function. The spectrum of the Rayleigh process is given by the Jake’s model. The channel autocorrelation function, defined by $\phi_{\tau, l}(\tau) = \sigma_{\tau, l} \ J_l(2\pi f_d \tau)$, is only function of the time difference $\tau$, $f_d$ is the maximum Doppler spread, and $J_l$ is the zero order Bessel function of the first kind.

If we consider chip rate sampling and that the receiver is synchronized to a desired user, $k$, then the signature of $k_{th}$ user can be written as

$$\begin{align*}
h_k = S_k g_k,
\end{align*}$$

where

$$\begin{align*}
S_k = \begin{bmatrix}
s_k(0) & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
n_k(N-1) & 0 & \cdots & 0 \\
0 & \cdots & \cdots & n_k(0) \\
0 & \cdots & \cdots & \cdots & 0
\end{bmatrix}_{(N+q)(q+1)}
\end{align*}$$

$$\begin{align*}
g_k = [g_k(0) \ g_k(1) \ \cdots \ g_k(q)]^T.
\end{align*}$$

### 3. Blind Minimum Output Energy Receiver

A linear receiver estimates the desired signal at symbol interval $n$ as

$$\begin{align*}
\hat{d}_k(n) = \hat{a}_k(n) + j \hat{b}_k(n) = w_k^H r(n)
\end{align*}$$

where $w_k$ are the filter tap weights. The hard symbol decisions are given by

$$\begin{align*}
\hat{a}_k(n) = \text{sgn}(\hat{a}_k(n)) \quad \text{and} \quad \hat{b}_k(n) = \text{sgn}(\hat{b}_k(n)).
\end{align*}$$

The blind Minimum Output Energy detector was presented in [1] for an AWGN channel with no multipath fading. In the absence of multipath, $g_k = 1$ and $h_k = s_k$.

The filter aims to minimize the output variance from the filter, defined by

$$\begin{align*}
e_k = E \left[ \left\| \hat{d}_k(n) \right\|^2 \right] = w_k^H R w_k,
\end{align*}$$

where $R = E \left[ r(n)r^H(n) \right]$ is the autocorrelation of the input vector, subject to the constraint that the response to the desired user, $k$, is a constant, $w_k^H s_k = 1$. The tap weight vector can be decomposed into two orthogonal components, $s_k$ and $x_k$, such that

$$\begin{align*}
w_k = s_k + x_k.
\end{align*}$$

The optimum weight vector was shown to be

$$\begin{align*}
w_{k, opt} = R^{-1} s_k (s_k^H R^{-1} s_k)^{-1}
\end{align*}$$

In multipath channels, [2], the output variance is minimized subject to the constraint

$$\begin{align*}
S_k^H w_k = g_k.
\end{align*}$$

This algorithm assumed the receiver to be synchronized to the desired user and has knowledge of the maximum order of the channel, $q$. Using Lagrange multipliers, the optimum tap weight vector was shown to be
\[ w_{k,\text{opt}} = R^{-1} S_k \left( S_k^H R^{-1} S_k \right)^{-1} g_k \quad \text{Eq. (9)} \]

and the MOE from the filter is
\[ \varepsilon = w_{k,\text{opt}}^H R w_{k,\text{opt}} = g_k^H \left( S_k^H R^{-1} S_k \right)^{-1} g_k. \quad \text{Eq. (10)} \]

In [3], an RLS adaptive algorithm was presented to compute the minimum variance solution. It was shown that the channel vector, \( g_k \), could be estimated by maximizing the minimum output variance subject to the constraint that \( \|k_i\| = 1 \).

This max/min criterion maximizes the output signal power after the interference has been eliminated. Thus, \( g_k \) is estimated to be the eigenvector, \( g_{k,\text{min}} \), corresponding to the minimum eigen-value, \( \gamma_{\text{min}} \), of the matrix \( (S_k^H R^{-1} S_k) \). Thus for a blind receiver with no knowledge of the channel response, the optimum tap weights are given by
\[ w_{k,\text{opt}} = \gamma_{\text{min}} R^{-1} S_k g_{k,\text{min}}. \quad \text{Eq. (11)} \]

### 4. MOE RECEIVER WITH CONSTRAINED SURPLUS ENERGY

In this paper, we consider an MOE receiver for multipath channels with constrained surplus energy. The receiver proposed in [3] assumed correct knowledge of the maximum order of the channel, \( q \). If \( q \) is underestimated, then the set of constraints developed by Eq. (8) will suffer from a mismatch. Also, the maximum delay spread was assumed smaller than one symbol interval, or \( q \) is less than \( N \) for chip rate sampling.

The tap weights of Eq. (9) can be written as
\[ w_k = h_k + x_k, \quad \text{Eq. (12)} \]

where \( h_k \) and \( x_k \) are two orthogonal components. The energy of the tap weights is
\[ \|w_k\|^2 = 1 + \chi, \quad \text{Eq. (13)} \]

where \( \chi = \|x_k\|^2 \) is called the surplus energy and \( \|h_k\|^2 = 1 \). This surplus energy can cause signal cancellation as well as noise enhancement. A constraint is put on \( w_k \) to prevent the increase in surplus energy. A similar constraint was proposed in [1] for the MOE algorithm proposed for AWGN channels to combat signal mismatch. The surplus energy was constrained to be equal to \( \chi \) such that \( \chi_i \leq \chi < \chi_S \) where \( \chi_i \) and \( \chi_S \) are

the minimum values of surplus energy required to cancel out all the interference and the desired signal respectively.

Minimizing the output energy subject to the constraint that \( S_k^H w_k = g_k \) and \( \|w_k\|^2 = 1 + \chi \), we define the following cost function
\[ \varepsilon = w_k^H R w_k - \lambda_i [w_k^H S_k - g_k^H] + \lambda_2 [w_k^H w_k - (1 + \chi)] \]

The optimum set of filter tap weights are
\[ w_k^* = \frac{[R + \lambda_2 I]^{-1} S_k g_k}{S_k^H [R + \lambda_2 I]^{-1} S_k} \quad \text{Eq. (14)} \]

and the surplus energy is
\[ \chi = \|w_k^*\|^2 - 1 = \frac{g_k^H S_k^H [R + \lambda_2 I]^{-1} S_k g_k}{[S_k^H [R + \lambda_2 I]^{-1} S_k]^2} - 1. \quad \text{Eq. (15)} \]

Noting that \( \frac{[R + \lambda_2 I]^{-1} S_k g_k}{S_k^H [R + \lambda_2 I]^{-1} S_k} \) are orthogonal,
\[ \sum_{k=1}^{K} e_k^2 \frac{P d_k^2}{h_k h_k^H}, \]

means that both the constraint, \( \lambda_2 \), and the noise variance, \( \sigma_i^2 \), have the same role in determining the minimum variance solution. However, for a given value of \( (\sigma_i^2 + \lambda_2) \), the performance is worse for higher \( \sigma_i^2 \). When no constraint is put on the surplus energy, \( \lambda_2 = 0 \) and the solution is the same as in Eq. (9).

In case the maximum order of the channel is underestimated, the paths of delays larger than the estimated channel order, \( q \), can be considered as an additional interference. The surplus energy determines the amount of allowable mismatch between the signature of the desired user, \( h_k \), available to the detector and the actual user’s signature. This constraint further prevents the increase of surplus energy at high Signal to Noise Ratios (SNR), or small \( \sigma_i^2 \), which results in cancellation of the desired signal.

This algorithm can be implemented by using the RLS algorithm as follows:

\[ * \quad P(0) = \partial I; \quad \partial \] is a small positive constant
\[ * \quad k(n) = \frac{\lambda^{-1} P(n-1)r(n)}{1 + \lambda^{-1} r(n) P(n)} \quad \text{Eq. (16)} \]
\[ P(n) = \lambda^{-1} P(n-1) - \lambda^{-1} k(n) r^H(n) P(n-1) \]  

Eq. (17)

* \[ \Omega(n) = \frac{P(n)}{[I + P(n)[\lambda, I]]} \]

Eq. (18)

* \[ w_k(n) = \gamma_{\text{min}}(n) \]  

Eq. (19)

where \( P(n) \) is the estimated inverse of the exponentially weighted autocorrelation matrix of the input vector such that \( \hat{r}(n) = \sum_{i=1}^{\infty} \lambda^{n-i} r(i) r^H(i) \).

\[ P(n) = \hat{R}(n) \]  

A large value of the Lagrangian multiplier, \( \lambda_2 \), denotes a small value of \( \gamma \), where a small value denotes a large permissible value. For blind detection, a reasonable value of \( \lambda_2 \) is chosen and can be made equal to the noise power that gives acceptable Signal to Interference Ratio (SIR) in the absence of the explicit constraint, i.e. when \( \lambda_2 \) is zero.

5. BLIND TRAINING OF MMSE DETECTORS

The MOE solution converges to the MMSE solution. Adaptive MMSE detectors using training sequences have better performance but training sequences add a burden to the system and should be sent often to scan new changes in the channel or interference structure. The MOE detector is blind in the sense that it converges to the MMSE solution without using training sequences. Thus, instead of being adapted by training sequences, the MMSE detector could be trained using the symbol decisions from the blind MOE detector. The performance of the MMSE detector would depend on the reliability of the decisions from the MOE detector. The MMSE detector implemented with the RLS algorithm [4] and the same contents of the MOE filter is as follows:

* \[ e(n) = d_k^*(n) - \hat{d}_k(n) \]  

Eq. (21)

* \[ w_k(n) = w_k(n-1) + \kappa(n) e^H(n) \]

Eq. (22)

where \( w_k(0) = 0 \), \( w_k(n) \) is the vector of the RLS filter’s coefficients at iteration \( n \) and \( d_k^*(n) \) is equal to the hard symbol decisions of the blind detector during the training period and to the hard symbol decisions of the MMSE detector during the decision directed mode.

6. SIMULATION EXAMPLES

In this section, we consider simulation examples to demonstrate our approach, (CST MOE), and compare it to the recursive approach of [3], (MOE). The system has 6 users transmitting \( 10^6 \) QPSK symbols per second, equivalent to a bit rate of 2 Mega bit per second (Mbps). The spreading sequences are gold codes with a spreading gain of 31. To avoid averaging over the relative delays of the users, a synchronous system is assumed. The multipath channel has a maximum Doppler shift of 100 Hz. The channel has 3 paths. The path gains are normalized for unity gain. The channel adds zero mean AWGN. The estimated maximum order of the channel \( q \) is fixed to be 31 chips for all simulations. The BER is calculated over a frame of 2500 symbols or 5000 bits. \( \lambda_2 \) is set to 100 and the forgetting factor, \( \lambda \), to 1.

Furthermore, the MOE and the CST MOE detector outputs are used to train an RLS filter in the MMSE sense, using the same input filter contents and the generated matrix, \( P(n) \), and are denoted by (MOE-MMSE) and (CST MOE-MMSE) respectively. The RLS filter is trained for the first 300 symbols of an 2500 symbol frame then switches to a decision directed mode. The BERs are calculated by neglecting the first 300 training symbols.

Case A: The maximum path delays are less than one symbol interval with relative gains and delays of \([0, -3, -3]\) dB and \([0, 2, 25]\) chips respectively. The average Bit Error Rate (BER) averaged over the six users is plotted versus Eb/No (Energy per bit to noise PSD ratio) in Figure 1. It is noticed that at a high SNR the constrained algorithm is superior as it prevents cancellation of the desired signal. Cancellation of the desired signal occurs due to imperfect estimation of the channel vector, which results in a mismatch in the desired user’s
A Near Far (NF) scenario is also simulated where the relative amplitude gain of one of the users is varied. The average BER of the other five equal power users is plotted versus the interferer gain in Figure 2. Eb/No was fixed to 15 dB. At a high SIR, the unconstrained algorithm has poorer performance due to the increase in the value of the surplus energy, $\chi$, which leads to noise enhancement at the output of the receiver. However, at a lower SIR, the surplus energy of the constrained algorithm is not enough to cancel out the interference and thus the constrained algorithm has poorer performance. A lower value of $\lambda_2$ would give better results in case of higher interference.

Figure 1: BER Comparison for path delays = [0, 2, 25] chips

Case B: The previous set of simulations is repeated for the same conditions except that the channel introduces ISI where the relative path delays are [0, 2, 35] chips respectively. Due to under-estimation of the maximum order of the channel $q$, the actual signature of the desired user will have a non-zero projection on the estimated signal subspace at the receiver. As a result, cancellation of the desired signal occurs if the value of the surplus energy is allowed to increase. Figure 3 shows the average BER versus Eb/No. It is noticed that the unconstrained receiver has poorer performance at higher SNRs due to cancellation of the desired signal. Figure 4 shows the NF resistance at Eb/No = 20dB. Our constrained algorithm has better performance although the MMSE filter is sensitive to interferer gains.

Figure 2: NF resistance for path delays = [0, 2, 25] chips

Figure 3: BER Comparison for path delays = [0, 2, 35] chips

7. CONCLUSIONS

We proposed an MOE detector with constrained surplus energy for multipath channels. Simulations show that it has superior performance over unconstrained detectors at high Signal to Interference and Noise Ratios and in cases of imperfect estimation of the channel order or the desired signal subspace at the receiver. The RLS MMSE receiver could be trained by means of a blind detector, which
reduces the burden of sending training sequences. The performance of the RLS detector depends on the reliability of the training symbols and thus on the performance of the blind detector.

![Graph showing NF resistance for path delays = [0, 2, 35] chips]

Figure 4: NF resistance for path delays = [0, 2, 35] chips

REFERENCES


