

# CYCLIC MINIMUM VARIANCE DETECTION OF MULTIRATE DS-CDMA IN MULTIPATH FADING CHANNELS

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## ABSTRACT

A cyclic minimum variance detector is presented for the blind detection of the higher data rate signals in a multirate asynchronous Direct Sequence Code Division Multiple Access (DS-CDMA) system and which are signaled over a multipath Rayleigh fading channel. The interference viewed by a higher data rate symbol will be periodic in nature due to the presence of a lower data rate symbol, which spans multiple higher data rate symbols. The detector applies a cyclic detection rule along with joint channel estimation. The convergence and tracking properties of the cyclic detector as well as the Bit Error Rate (BER) characteristics are illustrated using simulation examples.

## I. INTRODUCTION

Future third generation Direct Sequence Code Division Multiple Access (DS-CDMA) systems are required to support a variety of multimedia applications with different data rates. Different access schemes have been developed for multirate DS-CDMA such as the Multi-Code (MC), the Variable Spreading Length (VSL), and the Variable Chip Rate (VCR) access schemes [1][2]. Conventional detectors implemented in current DS-CDMA systems treat the Multiple Access Interference (MAI) due to other users as noise. Multiuser Detection (MUD) aims to suppress the MAI. Blind detectors, which do not need training sequences, such as the blind Minimum Output Energy (MOE) detector were proposed [3]. This detector was modified to operate in multipath channels [4] and adaptive algorithms for its implementation were discussed [5]. Several detectors were proposed for multirate CDMA in synchronous channels.

In this paper, we propose a cyclic blind minimum variance detector for the detection of multirate users using the VCR or the VSL access schemes in asynchronous multipath channels. We extend the work of [6], which assumes that the detector has perfect knowledge of the desired user's signature and assumes an Additive White Gaussian Noise (AWGN) channel. The cyclic detection rule is applied to accommodate for the periodic interference viewed by the higher rate users due to a lower data rate user whose data symbol and thus its corresponding interference spans multiple higher data rate symbols. Our proposed detector also forms joint channel estimation.

## II. MULTIRATE SYSTEM MODEL

For simplicity, we consider a dual rate asynchronous DS-CDMA system. The ratio between the high data rate and the low data rate is  $M$  such that  $T_s^{(l)} = MT_s^{(h)}$  and  $T_s^{(l)}$  is the symbol duration such that the superscripts  $(l)$  and  $(h)$  refer to the LR and HR users respectively. The received baseband signal model can be written as

$$r(t) = \sum_{n=-\infty}^{\infty} \left( \sum_{k=1}^{K^{(l)}} r_k^{(l)}(n) + \sum_{k=1}^{K^{(h)}} r_k^{(h)}(n) \right) + z(t), \quad (1)$$

where  $K^{(l)}$  and  $K^{(h)}$  are the number of the LR and HR users respectively and  $z(t)$  is an AWGN with zero mean and variance  $\sigma_z^2 = N_o/2$ . The received signal due to user  $k$  sampled at a multiple  $U$  of the chip rate is

$r_k^{(l)}(n) = \sum_{m=-\infty}^{\infty} A_k^{(l)} d_k^{(l)}(m) h_k^{(l)}(n - mN^{(l)}U)$ , where  $A_k^{(l)}$  is the received amplitude of user  $k$ ,  $d_k^{(l)}(n) = a_k^{(l)}(n) + jb_k^{(l)}(n)$  is the  $n_{th}$  QPSK symbol of the  $k_{th}$  user and is uncorrelated within and among users. The  $k_{th}$  user's normalized spreading sequence is  $s_k^{(l)}$ ,  $N^{(l)}$  is the spreading gain,  $g_k^{(l)}(n)$  is the sampled vector of the multipath channel impulse response of unity gain and maximum order  $q^{(l)}$  and  $q^{(l)} = \lceil T_m / T_c^{(l)} \rceil$ , where  $\lceil \cdot \rceil$  denotes upper integer,  $T_m$  is the delay spread, and  $T_c^{(l)}$  is the chip duration. Dropping the superscript,  $(\cdot)$ , the overall desired user's signature,  $h_k$ , such that  $\|h_k\| = 1$ , is expressed as

$$h_k = S_k g_k, \quad (2)$$

where  $S_k = \begin{bmatrix} s_k(0) & & & 0 \\ & \ddots & & 0 \\ s_k(N-1) & & & 0 \\ 0 & & & s_k(0) \\ 0 & & & \vdots \\ 0 & & & s_k(N-1) \end{bmatrix}_{(N+q) \times (q+1)}$  and  $g_k = [g_k(0), g_k(1), \dots, g_k(q)]^T$ .

For linear detection, the soft and hard outputs of a filter having a tap weight vector  $w_k^{(i)}$  and filter contents  $r^{(i)}(n)$  are  $\hat{d}_k^{(i)}(n) = w_k^{(i)H}(n)r^{(i)}(n)$  and  $\tilde{d}_k^{(i)}(n) = \tilde{a}_k^{(i)}(n) + j\tilde{b}_k^{(i)}(n)$  respectively, where  $\tilde{a}_k^{(i)}(n) = \text{sgn}(\hat{d}_k^{(i)}(n))$  and  $\tilde{b}_k^{(i)}(n) = \text{sgn}(\hat{b}_k^{(i)}(n))$ .

### III. MMSE DETECTION OF THE HIGH RATE USERS IN MULTIPATH CHANNELS

A high rate user views an interference that is cyclic in nature and has a period of  $M$  high rate symbols. Instead of having an optimum weight vector achieving the MMSE solution, a HR user has a set of  $M$  weight vectors to be employed periodically to achieve the cyclic MMSE solution [6]. Considering the high rate users and dropping the superscript,  $(h)$ , for the ease of notation, the true cyclic MMSE solution for the periodic tap weight vector is

$$w_k(p) = (R'(p))^+ J_k; \text{ for } p = 0, 1, \dots, M-1, \quad (3)$$

where  $R'(p) = E(r_p(n)r_p(n)^H)$  is the covariance matrix of the received vector such that  $r_p(n) = r(n)$  if  $p = \text{mod}(n, M)$ , and  $J_k = A_k h_k$  is the cross-correlation between the received vector and the desired solution..

### IV. A BLIND CYCLIC DETECTOR FOR THE HR USERS IN MULTIPATH CHANNELS

In this section, we propose a blind minimum variance detector for the cyclic detection of the high rate users transmitting over a multipath fading channel in a multirate DS-CDMA system. The soft output from the cyclic detector is

$$\hat{d}_k(n) = \hat{w}_k^H(n, i)r(n) \quad (4)$$

where  $\hat{w}_k(n, i) = \hat{w}_k(n, \text{mod}(n, M))$  is the estimate of the optimum weight vector,  $w_k(i)$ , for  $i = (0, 1, \dots, M-1)$ , after receiving  $n$  vectors. The receiver is assumed synchronized to the desired user and  $r(n)$  is of length  $\tilde{N}^{(h)} = (N^{(h)} + q)$  for chip rate sampling. The detector could be easily extended to the case when it is not synchronized to the desired user by letting its observation window span multiple symbols. The  $M$ -periodic tap weight vector,  $\hat{w}_k(n, i)$ , is expressed in terms of

its Discrete Time Fourier Series (DTFS) coefficients [6] as  $\hat{w}_k(n, i) = \sum_{q=0}^{M-1} W_k^n(q) \exp\left(j \frac{2\pi q i}{M}\right)$ , where  $\{W_k^n(q)\}_{q=0}^{M-1}$  is the  $q$ th Fourier coefficient. The  $(M\tilde{N}^{(h)} \times 1)$  vector,  $\tilde{W}_k^n = [W_k^{nT}(0), W_k^{nT}(1), \dots, W_k^{nT}(M-1)]^T$ , contains the  $M$  augmented Fourier coefficients. The detector minimizes the output energy subject to the constraint that the response to the desired user's signature is constant. We define  $\hat{R}(n) = \frac{1}{n} \sum_{i=1}^n \lambda^{n-i} r(i)r^H(i)$  as the exponentially weighted input covariance matrix,

$\lambda$  is the forgetting factor,  $\tilde{r}(i) = \left[ r^H(i), r^H(i) \exp\left(j \frac{2\pi i}{M}\right), \dots, r^H(i) \exp\left(j \frac{2\pi(M-1)i}{M}\right) \right]^H$ ,  $\tilde{R}(n) = \sum_{i=1}^n \lambda^{n-i} \tilde{r}(i)\tilde{r}^H(i)$ ,  $\tilde{S}'_k = I_M \otimes S_k$  which is an  $(M\tilde{N}^{(h)} \times M(q+1))$  matrix where  $\otimes$  denotes a Kronecker product,  $E_1 = [I_{q+1}, 0, \dots, 0]^T$  is an  $(M(q+1) \times (q+1))$  matrix, and  $\hat{w}_k^H(n, i)r(i) = \tilde{W}_k^{nH} \tilde{r}(i)$ . Defining  $C$  as constant, the constraint and its equivalent in terms of the DTFS coefficients for minimizing the filter's output energy could be expressed as,

$$\begin{cases} \hat{w}_k^H(n, i)\hat{R}(n)\hat{w}_k(n, i) = \min \\ h_k^H \hat{w}_k(n, i) = C \quad \forall i = 0, 1, \dots, M-1 \end{cases} \quad \begin{matrix} = \\ = \end{matrix} \quad \begin{cases} \tilde{W}_k^{nH} \tilde{R}(n)\tilde{W}_k^n = \min \\ \tilde{S}'_k{}^H \tilde{W}_k^n = E_1 g_k \end{cases} \quad (4)$$

Using Lagrange multipliers, the optimum DTFS coefficients are given by  $\tilde{W}_k^n = \tilde{R}^{-1}(n)\tilde{S}'_k(\tilde{S}'_k{}^H \tilde{R}^{-1}(n)\tilde{S}'_k)^{-1} E_1 g_k$ .

For blind detection with no perfect knowledge of the channel response, the channel vector,  $g_k$ , could be estimated [4]. The filter's minimized output energy is maximized subject to the constraint that  $\|g_k\| = 1$  or  $\|E_1 g_k\| = 1$ . This maximizes the

signal energy after suppressing the interference. The filter's minimum output variance is  $\zeta_{\min} = \tilde{W}_k^n H \tilde{R}(n) \tilde{W}_k^n H$ . Maximizing  $\zeta_{\min}$ ,  $g_k$  is estimated to be the eigen-vector,  $\tilde{g}_k$ , corresponding to the minimum eigenvalue,  $\gamma_{\min}$ , of the  $((q+1) \times (q+1))$  matrix,  $E_1^H (\tilde{S}_k'^H \tilde{R}^{-1}(n) \tilde{S}_k') E_1$ . The required DTFS coefficients are thus given by,

$$\tilde{W}_k^n = \gamma_{\min} \tilde{R}^{-1}(n) \tilde{S}_k' E_1 \tilde{g}_k \quad (5)$$

The weights,  $\hat{w}_k(n, i) = (e_2^H \otimes I_{\tilde{N}(h)}) \psi(i) \tilde{W}_k^n$ , are constructed from their Fourier coefficients where  $e_2 = [1, 1, \dots, 1]^T$  and

$$\psi(i) = \text{Diag} \left[ 1, \exp\left(j \frac{2\pi i}{M}\right), \exp\left(j \frac{2\pi(2)i}{M}\right), \dots, \exp\left(j \frac{2\pi(M-1)i}{M}\right) \right]^T \otimes I_{\tilde{N}(h)}. \text{ With a proper assumption of } q, \tilde{g}_k \text{ forms channel estimation.}$$

## V. ADAPTIVE IMPLEMENTATION OF THE CYCLIC ALGORITHM

The recursive algorithm, with  $P(n)$  estimating  $\tilde{R}^{-1}(n)$ , could be summarized as follows:

Initialize  $P(0) = \theta^{-1} I$ , where  $\theta$  is a small positive constant. For  $n = 1, 2, \dots$ , compute:

- (1)  $\beta(n) = \lambda^{-1} P(n-1) \tilde{r}(n)$
- (2)  $\kappa(n) = [1 + \tilde{r}^H(n) \beta(n)]^{-1} \beta(n)$
- (3)  $P(n) = \lambda^{-1} P(n-1) - \lambda^{-1} \kappa(n) \tilde{r}^H(n) P(n-1)$
- (4) Find  $\gamma_{\min}$ , the minimum eigenvalue of  $E_1^H (\tilde{S}_k'^H P(n) \tilde{S}_k') E_1$ , and its eigen-vector  $\tilde{g}_k(n)$ .
- (5)  $\hat{w}_k(n, i) = \gamma_{\min} (e_2^H \otimes I_{\tilde{N}(h)}) \psi(i) P(n) \tilde{S}_k' E_1 \tilde{g}_k, \quad \forall i = 0, 1, \dots, M-1$  (6)

This algorithm's complexity is linear in  $M$  and is only  $M$  times greater than the non cyclic algorithm in [5].

## VI. SIMULATION EXAMPLES

We study the convergence of our proposed algorithm (Cyclic MOE) to the optimum cyclic MMSE solution. The convergence is interpreted by means of the average of the correlation coefficients between the  $M$  estimated tap weight vectors in (6) and their corresponding optimum MMSE cyclic vectors calculated over the whole frame by (3). The convergence properties are compared with those of the blind MOE RLS (MOE) algorithm in [5], which does not assume cyclic interference. The convergence of the channel estimate and BER characteristics are also studied.

We consider a multirate system using the VCR access scheme with  $M=3$ , 3 HR users and 3 LR users. Gold codes with a spreading gain of 31 are employed. A multipath Rayleigh fading channel is assumed. The maximum Doppler frequency is  $f_d=14.5$  Hz. The high rate user transmits  $10^6$  QPSK symbols per second. The forgetting factor of all the RLS algorithms is given by  $\lambda=0.9995$ . The amplitude gain of the HR and LR users is represented by  $A_H$  and  $A_L$  respectively. The estimated order of the channel (maximum delay spread) used in the compared algorithms is  $q=5$  HR chips.

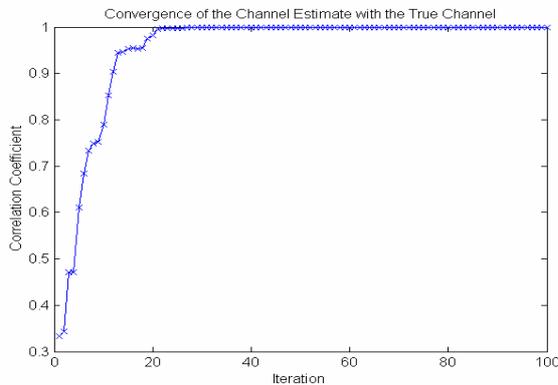


Fig. 1. Convergence of the Channel Estimate with the True Channel for Case A.1.

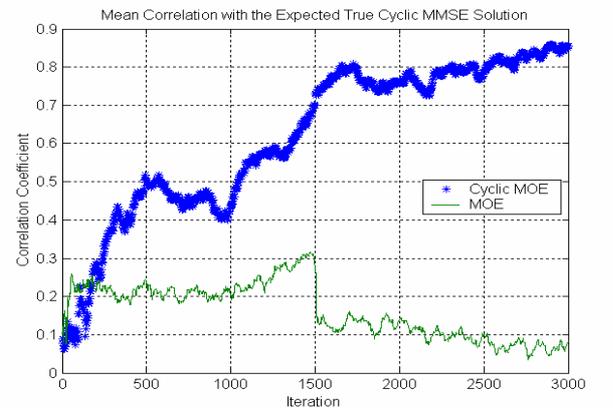


Fig. 2. Convergence with the Optimum Cyclic Solution for Case A.2.

(A) A channel with 2 paths is assumed with SNR=20 dB and relative delays and amplitude gains of [0, 3] HR chips and [0, -3] dB respectively. In case (A.1),  $A_L=20$  dB and  $A_H=0$  dB, where the cyclic interference is emphasized by increasing the amplitude gain of the LR users. The convergence of the channel estimate with the true channel is illustrated in Fig. 1. In case (A.2), 3 new LR users with amplitude gain of 20 dB enter the channel at iteration 1500. The old LR and HR users have  $A_L=0$  dB and  $A_H=0$  dB respectively. Convergence with the theoretic MMSE solution is shown in Fig. 2.

(B) We compare our proposed cyclic algorithm (Cyclic M) with that of [6], which is referred to as (Cyclic A). Both algorithms have knowledge of the desired user's spreading code only. The same user and channel parameters in case A.1 are assumed. In Fig. 3, the average BER of the HR users is plotted. The Near Far resistance is simulated where the average BER of the HR users, having a unity amplitude gain, is plotted versus the amplitude gain of the LR users in Fig. 4.

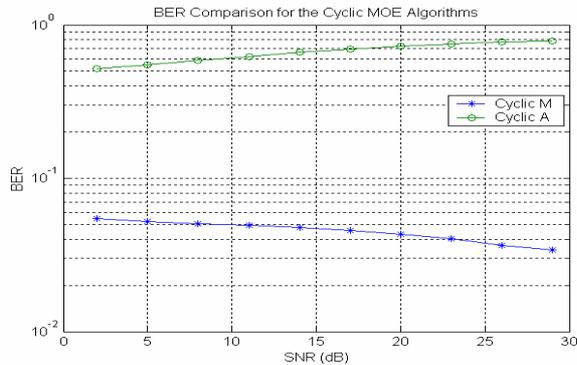


Fig. 3. BER Comparison of the Cyclic Algorithms.

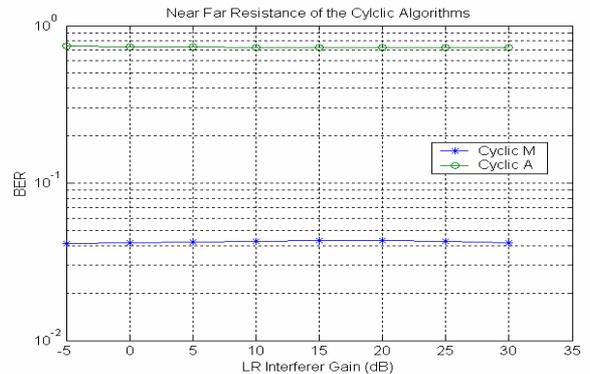


Fig. 4. Near Far Resistance of the Cyclic Algorithms.

Our proposed algorithm correctly models the channel's periodic interference structure, and thus has higher correlation coefficients with the optimum cyclic solution than the static MOE algorithm does. It has good tracking properties where the solution converges to the true solution even if new LR users, adding to the cyclic properties of the channel, enter the channel in the middle of the frame. Our proposed cyclic algorithm could adapt to the new cyclic interference where as the conventional static MOE algorithm could not. With a only proper estimate of the maximum channel order,  $q$ , our algorithm has superior performance over that of [6], which assumes perfect knowledge of the desired user's signature leading to a mismatch between the available signature at the receiver and the actual user's signature due to the multipath channel, and thus cancellation of the desired signal and signal degradation at the output of the receiver.

## VII. CONCLUSIONS

We proposed a blind cyclic minimum variance algorithm for the detection of the higher rate data signalled over multipath channels in a multirate DS-CDMA system. A recursive implementation of the algorithm was proposed. The algorithm converges to the optimum cyclic MMSE solution and has good tracking properties. With the correct assumption of the maximum order of the channel, the algorithm is superior to previously proposed cyclic algorithms which assume perfect knowledge of the desired user's signature.

## VIII. REFERENCES

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