Prologue:
The Hamming Code
Summary

Of the 35 patterns of three erasures:

- 25 are correctable with the simple algorithm
- 3 more are correctable with the complex algorithm
- 7 are uncorrectable by any algorithm
In General

Of the $\Theta(n^3)$' patterns of three erasures:

- $\Theta(n^{2.322})$ are correctable with the "message-passing" algorithm
- $\Theta(n^3)$ are correctable with ML algorithm
- $\Theta(n^2)$ are uncorrectable
1948: The Information Age Begins
Claude E. Shannon
1916--2001

“The fundamental problem of communication is that of reproducing at one point either exactly or approximately a message selected at another point.”

To solve the fundamental problem, Shannon created a branch of applied mathematics which is today called Information Theory...
Information Theory 1101
Entropy

Absolute entropy $H(X)$ measures the amount of uncertainty about $X$. Relative entropy $H(X|Y)$ measures the uncertainty in $X$ after $Y$ is observed.

\[
H(X) = - \sum_x p(x) \log p(x)
\]
\[
H(X|Y) = - \sum_{x,y} p(x, y) \log p(x|y)
\]
Mutual Information

Mutual Information $I(X;Y)$ measures the amount of information the event $Y$ provides about the event $X$

$$I(X;Y) = H(X) - H(X|Y)$$
Capacity

The capacity of a channel is the highest rate (typically measured in bits per second) at which reliable communication over the channel is possible.

\[ C = \max_X I(X; Y) \]
Compressibility

The Compressibility function $R(\delta)$ is the minimum number of bits per second required to communicate the source output with “acceptable distortion” $\delta$.

$$R(\delta) = \min_{Y : |X - Y| \leq \delta} I(X; Y).$$
Shannon’s Equations

\[ H(X) = - \sum p(x) \log p(x) \]

\[ I(X; Y) = H(X) - H(X|Y) \]

\[ C = \max I(X; Y) \]

\[ R = \min I(X; Y) \]
Channel Capacity for the Space Channel

\[ \frac{E_b}{N_0} > \ln 2 \]
The Shannon Limit is approximately -1.59 dB.
1962: Planetary Exploration Begins at JPL
The Early Years: No Coding:

- Mariner 2, 1962
  - Venus Flyby
- Mariner 4, 1965
  - Mars Flyby
  - First close-up photographs of another planet.
- Mariner 5, 1967
1969: Channel Coding Begins at JPL
(32,6) Biorthogonal Code + "Green Machine" Decoding

- Mariners 6, 7 (1969)
- Mars Flyby
- Mariner 9 (1971)
- Mars Orbit
(32,6) Biorthogonal Code/
“Green Machine” Decoding

- Mariner 10, 1973-1974
- Mercury and Venus
- Viking Mars Landers, 1976
- Mars’ Surface

The (8,4) biorthogonal code
\[ \log_{10} P_b \]

vs.

\[ \frac{E_b}{N_0}, \text{dB} \]

- Shannon Limit: -1.59 dB
- Uncoded
- (32,6) Biorthogonal

-6 to -1 on the vertical axis
-1.0 to 11.0 on the horizontal axis
\( K = 7, \ R = 1/2 \) Convolutional Code + Viterbi Decoding

- Voyagers 1&2 (1977-- )
- "Grand Tour"
- Mars Global Surveyor (1997- )
$\log_{10} P_b$ vs $E_b/N_0$, dB

- **Uncoded**
- **(32,6) Biorthogonal**
- **(7, 1/2) Conv.Code**

**SHANNON LIMIT**
- $-1.59$ dB
K = 7, R = 1/2 Convolutional Code
+(255,223) Reed-Solomon

- Voyagers 1&2 (1977--)
- “Grand Tour”
- Mars Global Surveyor (1997--
$\log_{10} P_b$

$\frac{E_b}{N_0}, \text{dB}$

- Uncoded
- (32,6) Biorthogonal
- (7, 1/2) Conv.Code
- (7, 1/2) + (255,223) RS
K = 15 Convolutional Code + RS with Big Viterbi Decoding

- A Sea of Troubles
- Sojourner
K = 15 Convolutional Code + RS + Big Viterbi Decoding (continued)

- Cassini (1997 ---- )
- Huygens Titan Probe, 2005
- Spirit and Opportunity
\begin{align*}
\log_{10} P_b &\quad E_b/N_0, \text{ dB} \\
\text{Uncoded} &\quad (32,6) \text{ Biorthogonal} \\
(7, 1/2) \text{ Conv.Code} &\quad (7, 1/2) + (255,223) \text{ RS} \\
(15,1/6) + (255,223) \text{ RS} &
\end{align*}

\text{SHANNON LIMIT} -1.59 \text{ dB}
Turbo Codes

Claude Berrou

Alain Glavieux
A Brave New World: Turbo Codes

- **Messenger to Mercury (APL Mission: 2004--2011)**

- **Mars Reconnaissance Orbiter (Aug 2005 Launch)**

  Both use (8920, 1/6) CCSDS turbo code
\[
\log_{10} P_b
\]

\[E_b/N_0, \text{dB}\]

- Shannon Limit: -1.59 dB

- Uncoded
- (32,6) Biorthogonal
- (7, 1/2) Conv.Code
- (7, 1/2) + (255,223) RS
- (15,1/6) + (255,223) RS
- (8920, 1/6) Turbo
Back to the Future:
LDPC Codes

- Mars Telecomm Orbiter 2010
- And Beyond!
How We May Appear too Future Generations

Claude Shannon — Born on the planet Earth (Sol III) in the year 1916 A.D. Generally regarded as the father of the Information Age, he formulated the notion of channel capacity in 1948 A.D. Within several decades, mathematicians and engineers had devised practical ways to communicate reliably at data rates within 1% of the Shannon limit . . .

Encyclopedia Galactica, 166th ed.
EPILOGUE
“The fundamental problem of communication is that of reproducing at one point either exactly or approximately a message selected at another point.”

“Frequently the messages have meaning”